

Manuel Bremer

Concept and Analysis

A Study in the Theory of Concepts
and Analytic Metaphilosophy

λογος

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Logos Verlag Berlin GmbH
Comeniushof, Gubener Str. 47,
10243 Berlin
Tel.: +49 (0)30 42 85 10 90
Fax: +49 (0)30 42 85 10 92
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[T]o arrive at a clear understanding of the most general features of our conceptual structure, as it exists in fact – whether or not it is possible to demonstrate the necessity of those features – is a sufficient task for any philosopher, however ambitious.

(Peter Frederick Strawson)

Preface

The title of this book may seem to err in several respects: concepts are today properly studied in the cognitive sciences (especially linguistics and cognitive psychology), 'analysis' is a very vague covering term for supposedly quite distinct methods, at least one of which, namely 'conceptual analysis', has had its share of bad press in the last 50 years, and, finally, the subtitle distinguishes between philosophy and meta-philosophy, where philosophy as the proverbial meta-science cannot be distinguished from its meta-science.

There is some truth in these accusations. Nonetheless the very aim of this book is to set out in which respects concepts are properly studied in philosophy, what methodological role the study of concepts has in philosophy's study of the world, why there are several viable methods of analysis and even conceptual analysis has its place here. I do not like the talk of 'meta-philosophy' myself, but many of the considerations in this book nowadays are placed under that headline, so I just followed common – although somewhat foolish – practice.

The book starts with some bold theses in favour of a representationalist theory of meaning and concepts. They have to be stated so boldly at the beginning as they serve as the background for the discussion in the following chapters, and as defending them in detail required some other and much longer book. In contrast to paradigmatic ordinary language philosophy I endorse a representationalist theory of meaning and concepts, thus agreeing with many of its critics in philosophy and the cognitive sciences. In contrast to many of these critics and supposedly the majority of cognitive scientists I endorse the viability of conceptual analysis as one method of philosophy. Thus, whereas I hope to combine insights from both camps the position developed may earn the scorn of zealots of both camps, not to mention the contempt of the small minority of those rejecting both representationalism and conceptual analysis.

The representationalist theses reject a Fregean account of meaning, at least in some understanding of it. The second chapter, however, reflects on Frege's theory of concepts, because Frege's theory of concepts was one strand that inaugurated analytic philosophy. Frege's theory of sentential unity has barely been superseded, and the problems arising from Frege's understanding of concepts are still alive.

Frege's theory and the related problems in Frege's logic as in the *Grundgesetze der Arithmetik* (most famously the antinomy known as 'Russell's Paradox' going back to Frege's 'Basic Law V') lead over to the third chapter, which considers the proper approach to our concept of logic and the issue of psychological and ontological realism in logic and mathematics.

The fourth chapter continues on this topic and argues that ordinary language cannot express real truth-value gaps, and thus that its logic cannot prevent antinomic reasoning by recourse to truth-value gap semantics or logics.

The fifth chapter as the central chapter of the book starts by reconsidering the approach and the idea of ordinary language philosophy and its understanding of conceptual analysis. Although ordinary language philosophy cannot be the whole of analytic philosophy, given what was said in the preceding chapters and given the methodological claims made therein, a proper understanding of conceptual analysis turns out to be one part of analytic philosophy. The chapter starts with a general discussion of ordinary language philosophy, but proceeds then by a methodological overview and attempts to engage in some ordinary language philosophy concerning epistemological topics.

The sixth chapter differs from the rest of the book in three crucial respects. First, the chapter deals with the history of philosophy, whereas in the second chapter the reflections on Frege aim at systematic theses on sentential unity and concepts. Second, the very intelligibility of the chapter's topic (i.e. Hegel's Dialectics) stands in question, and the chapter may contain another failing attempt to grasp it. Third, the chapter is in German, because one might have to consult the original quotes in any case, and supposedly anyone with a serious interest in Hegel will be able to read German, much more so than with those having a historical interest in Frege or Wittgenstein. I included the chapter nonetheless, because my attempts to understand what is going on in Hegel's *Wissenschaft der Logik* lead to an interpretive approach that makes Hegel look like pursuing a pure form of conceptual analysis (i.e. one which wants to circumvent the traps of ordinary language). The chapter sees two methods at the heart of Hegel's Dialectics: (i) a type of connective analysis which works with conceptual contrasts, and (ii) an increase in complexity by a form of creative synthesis (i.e. something close to the method of the same name pursued by Russell, a one-time Hegelian, and other analytic philosophers).

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A Representationalist Theory of Concepts and Meaning

These theses aim at a *systematic* account of meaning. These theses on language and mind, and the ontological assumptions should be supported by the overall picture developing following and applying these theses in an account of cognition and language. Here they serve as a background to the discussion of other approaches in this book.

§1 Beliefs

A belief is a relation to a propositional *representation* in some *natural or mental language* (and thus indirectly to the *informational content* of this representation). The mind/brain is best understood following some computational and *representational theory of the mind* (following Fodor 1975 and Pylyshyn 1984). An occurring belief processing a propositional representation employs this representation, but relates the subject of the belief (or another propositional attitude) to the content of the representation. The informational content can be specified as an eternal sentence.

Even if not all concepts are representational (i.e. refer to parts of reality) the representational function is fundamental. Expressing pro-attitudes and most illocutionary acts depend on employing sentences and concepts because of their representational content. Communication depends on representation.

§2 Eternal Sentences and Referential Content (Part 1)

Assertoric sentences are evaluated as being true or false corresponding to their referential content being realized in a fact. Truth bearers are, thus, token sentences. Of course only declarative sentences uttered in a situation of usage make an assertion. Within these some contain temporal or otherwise indexical expressions which are anchored to referents given the situation of usage. Assertion of such an indexical sentence yields an *eternal sentence* by substituting for the indexical expressions other expressions which refer to the entities the indexicals are anchored to in that given situation of utterance. These eternal sentences express the *content* of the assertion made. Their own *informational content*, constituted by the referents of their constituent expressions, if they have referents, captures the informational content of the assertion made in that context. Truth bearers are, properly speaking, eternal declarative sentences, either asserted themselves or going back to an asserted indexical declarative

sentence. Indirectly one may consider the indexical sentences themselves as bearers of truth.

§3 Concepts as Core of Verbal Meaning

Two expressions of a natural language have the same meaning if they point to the same concept as their core meaning in their lexical entry. [The lexical entry specifying the meaning of an expression or morpheme will contain besides a pointer to the concept other elements like: syntactic and phonetic features, categorical features, c-selection and θ -roles, if any, links to other concepts.]

A concept is a type of *Language of Thought* (LoT) expression. Beliefs are compositionally constituted by tokens of such concepts (cf. Fodor 1998). Two LoT-expressions cannot have the same meaning, because LoT-expressions do not have meaning at all, they *are* the essential ingredient in the meaning of natural language expressions. Two LoT-expressions having exactly the same referential content are *identical*, since LoT-expressions are configurations in the mind/brain hooked to parts of reality and these pathways of hooking up with reality single out one configuration referring to a property in question. A token of a LoT-type is, in case it refers, *directly referential*: as LoT-tokens are representations themselves, no other *representation* mediates their hooking up to reality (in some way). They need no Fregean ‘sense’ to mediate their relation to their referents.

Two people share a concept in case they share the capacity to form a LoT-token of a LoT-type which refers to a property. By tokening such LoT-symbols in a situation of concept employment they are able to both hook up to the same present (or deferred present) instance (trope) of the property, thus sharing conceptual content. If they employ tokens of the same type they also share the representational means to represent that content (i.e. their representations are not just extensionally equivalent, but are equivalent in a way that resembles and substitutes for sameness of Fregean ‘sense’).

A natural language expression has a meaning, and by this (indirectly) referential content (as the concept [LoT-expression type] being the core of that meaning has referential content), *and* its *syntactic* properties. A LoT-expression has referential content and its syntactic properties – and nothing else. So a complex LoT-expression also has referential content and syntactic features – and nothing else. The referential content of the complex LoT-expression derives compositionally from the referential content of its constituting LoT-expressions.

Consider now

- (1) The teacher of Alexander wrote the *Analytiks*.
- (2) The most famous pupil of Plato wrote the *Analytiks*.

Their referential content contains different properties. Somebody can believe (1) but deny (2) for the well-known reason that thus different LoT-expressions are involved. So the mere syntactic differences between either the natural language or the LoT-expressions involved accounts for this type of phenomenon, usually associated with (Fregean) 'sense'. This holds even if proper names had no descriptive content, as two different names may point to different LoT-labels, which again differ syntactically.

As (1) and (2) have different referential content they refer to different facts. It happens that both have the same truth value (in the actual world). In (1) and (2) only co-referential expressions can be substituted for each other to preserve referential content. Especially “teacher of Alexander” and “most famous pupil of Plato” cannot be substituted for each other without changing referential content. The two expressions can be interchanged preserving the truth value of the sentences. As the truth value of a sentence is its semantic evaluation and not its reference that interchangeability does not make the two expressions co-referential. What do they share? The simplest answer, the answer which does not introduce new entities, is to say that these expressions are 'extensionally equivalent'. 'Extensional' is used here in the common usage of elementary logic: affecting the truth value respectively depending on truth value only. In contrast two expressions sharing referential content are 'referentially equivalent'. So referential equivalence is a tighter relation than extensional equivalence, but to distinguish these two relations no Fregean 'senses' have to be introduced.

[To proceed from here to 'intensionally equivalent' we have to enter semantic two-dimensionalism (cf. Chalmers 1996: 56-71), as, although referential content is identical to itself in all possible worlds (models), the possession of some referential content by an (LoT-)expression depends on empirical contingencies.]

§4 Definitions

That two expressions share their meaning need not imply that they share all their logical properties, as these depend on syntactic features as well. Syntactic differences account for differences in a *derivational (i.e. mechanical) system*, no further ingredient of 'sense' is needed for this. Nominal definitions serve the purpose of facilitating derivations by chunking content in more feasible representations. *Definiens* and *definiendum* share their meaning and informational content, they can be interchanged and thus the more feasible syntactic features of the defined expression are exploited.

§5 Thoughts

The core of the meaning of a natural language sentence is the LoT-sentence built up compositionally from the conceptual content of the expressions which

make up the natural language sentence. This LoT-sentence may be dubbed 'thought'. As the building LoT-tokens have referential content, so the *thought* has referential content. The thought may be connected to a fact if its content, specifiable by an eternal sentence, is realized in reality. All synonymous sentences of a natural language expressing the same thought have the same informational content. As the LoT-types are hooked up with parts of reality and thus are *directly* referential, there is no 'mode of givenness' coming with thoughts, it seems. There are two ways 'modes of givenness' may enter in this picture:

- (i) we process the natural language sentence in our mind and its very features (i.e. syntactic features) distinguish its way of presenting a content from other synonymous sentences, or
- (ii) we process the thought and although LoT-expressions need not be given to consciousness directly, again syntactic features of LoT-expressions may be relevant in distinguishing a specific LoT-sentence from synonymous ones if such exist at all.

Our picture thus involves natural language sentences (i.e. syntactic entities), LoT-sentences (a.k.a. thoughts) as the core of their meaning, and referential content connected to the components of the thought.

The thought and other LoT-items involved in the realization of inner and outer speech make up a representational whole. The meaning of a sentence is part of the representational whole which is processed when the sentence is spoken (overtly or in inner speech). Thoughts are not abstract objects like Fregean 'senses', but LoT-representations. All which is done by Fregean 'sense' is done and accounted for by the sentences (be it natural language or LoT-sentences) and their syntactic features. We think in sentences and we think about their content.

§6 Conceptual Links

Analytic dependencies and prototypical justification rules are closely associated to but need not be part of the core meaning of a sentence. *Conceptual atomism* (cf. Fodor 1998) claims that many if not most concepts cannot be decomposed into a set of conceptual parts or features thus that this set of features is not just necessary but also provides a sufficient analysis of the concept, thus that the conjunction of the features is equivalent to the concept in question. Of course there are lots of concepts that are derived compositionally from these atomic concepts, and these derived concepts can, obviously, be decomposed again. As many concepts are atomic this can be easily stated in a disquotational theory of truth. Having at some level of presentation a representation of this theory is part of semantic knowledge: *internal semantics* (cf. Chomsky 2005, Larson/Segal 1995).

The articulation of concepts within some natural language, nonetheless, is inherently connected to question of justifying the use of some expression in a specific situation. Even if these justifications are not meaning constitutive they are part of what competent speakers got to know when they acquired their language. Even if possessing some concept does not require (in all cases) being able to verify the presence of one of its instances, and even if being a competent user of a word does not require being able to justify the employment of that expression under all circumstances, someone sometime has to be able to link the word to the concept and thus to situations of justified use. Even if a concept cannot be completely decomposed into constituent concepts conceptual links to other concepts or partial decomposition link the application of a concept to these other concepts which one may have mastery of and thus may use their applicability as *criteria* for the applicability of the concept being linked to them.

Further on, the possession of a manifold of concepts may involve conceptual links, which are stronger than empirical generalizations. Even if a concept does not analytically decompose into a set of constitutive concepts, used in phrasing a definition employing 'necessary' and 'sufficient conditions', a partial decomposition might be possible, as expressed in *conceptually* true implications. For instance, "Cora is a cat" implies "Cora is a mammal", however or not "() is a cat" can be decomposed. Possessing both concepts involves establishing or discovering links between them, which are stronger than the link between the first sentence and "Cora likes to chase laser pointer dots", which is highly probable given that Cora is a cat.

§7 Truth

A true sentence refers to a fact (being the referential content of the thought expressed by the sentence). A belief is true if and only if the eternal sentence yielded by the belief's representational component corresponds to a fact (i.e. a structured piece of reality containing the referents present and governed by the main relation present in the eternal sentence).

§8 Referential Content (Part 2)

Facts are part of reality. Facts are a *sui generis* ontological category besides objects. We may picture facts by expressions which build on expressions we use for sentences, but facts do not become sentences or objects by this. Chunks of reality can be referred to by singular terms (and thus be considered as objects) or by true sentences (and thus be considered as facts). In this sense 'fact' and 'object' are ontological categories (i.e. categories employed within our ontological framework) covering the same pieces of reality. That doesn't mean in any sense that there either aren't objects or aren't facts. A chunk of reality has structure, taken the structure into consideration we have a fact, otherwise an object. That we 'take into consideration' again doesn't mean that it

otherwise isn't there. Object talk refers to objects. Fact talk refers to facts. Some object talk and some fact talk can refer to the same chunk of reality. It is wrong to state that objects do not exist in reality, because we do not take into consideration (abstract away from) some structures present in them.

Properties ('universals' in some sense of that term) are abstract entities in the sense of being a structure or structural component of an object (a chunk of reality), i.e. abstract in as much as they have *no independent existence*. They are thus just the opposite of Platonic 'forms' ('abstract entities' in another sense). Them being abstract in this way doesn't make them non-existent, constructed by us and the like. We refer to them as structures of reality by our concepts. In that sense we have *access* to abstract entities! A general term is unsaturated *because* its referent is an abstract entity dependent on an object. The singular term referring to this object taken as argument of the general term in question yields a true atomic sentence.

The conceptual links between concepts or their respective (partial) decomposition may be founded in some of the (mereological or causal) relations governing the properties they refer to.

§9 Semantic Rules

To *know the meaning* of an expression is only partially explained by knowing the conventions governing its use. It is better to say that as far as we participate in these conventions of usage (and thus have at least implicit knowledge of them) we know the lexical entry of the expression (i.e. know of the link to a concept and its reference). Conventions of use establish and maintain the link between phonemes/graphemes of a type and the conceptual components of the lexical entry. The lexical entry itself covers semiotic features syntactical (in the broad sense of including phonetic features), semantic features and pointers to analytic dependencies and pragmatic markers. Conventions of usage thus do not exhaust the meaning of a word. They correspond broadly to the meaning of a word, so that we can come to understand the meaning of a word if we follow its usage. Which concept is linked to a word by a convention is by no means a trivial problem.

Concepts are not constituted by (semantic) rules, but *expressing* some concept by a specific *word* within some linguistic community requires rules and possibly shared knowledge of them. Conceptual content is not constituted by rules of usage, but rules of proper usage *trace* the applicability of concepts given the presence or deferred presence of their referents.

So, *identifying* the meaning of a word has to consider these rules, which by this are rules of meaning (semantic rules). Although knowing the conventions of usage is not necessary to understand a word's meaning exhibiting mastery of these conventions is *sufficient* to show one's understanding of the meaning.

§10 Metarepresentation and Indirect Contexts

A meta-representation in the *narrow* sense is a representation the content of which contains another representation. Quotations, (numerical) codings (like Gödel-numberings) or higher order beliefs are taken to be typical examples. Representations concerning our cognitive (representational) faculties may be taken as meta-representations in a *broader* sense. They do not contain individual other representations, but their content contains or refers to representational properties (i.e. either properties of individual representations in their function as representations or properties of some faculty inasmuch as they are invoked in the explanation of its representational function).

A justification of a claim α may invoke other representations of α 's level. α is then justified with respect to its level or citing properties of representations of that level (e.g. being an observational belief in seemingly normal conditions). A justification of a claim α may invoke beliefs about the proper workings of claims of α 's type (e.g. beliefs about the reliability of observation). In this case α 's justification is meta-representational in the broader sense. Judgements of coherence (say, of one belief or statement cohering with others) are meta-representational.

In semantics itself the question of meta-representation arises in several connections. Linguistic division of labour allows that we defer to the experts. This linguistic knowledge again is meta-representational because it has to quote the term it is knowledge about. Speakers also have to have some accessible, though often sub-doxastically used, knowledge of the semantic rules of their language. Updating one's description of the world in case of conflicting data or expectations includes meta-representations concerning proper usage.

A dispositional or sub-doxastic belief needs nothing besides an LoT-sentence. An occurring belief if it is accessible to consciousness involves some further representation (like a natural language sentence verbalized in inner speech) as LoT-sentences are neither phenomenally given nor immediately accessible as such. Even some sub-doxastic or dispositional beliefs may be tied to some specific way to express that belief (by mechanisms of memory or by limited expressive power of the cognitive system under discussion).

Meta-representations are vital in *de dicto* attitude attributions. In a *de dicto* report somebody stands in the relation of belief to a sentence either identical or at least synonymous to the sentence used (not mentioned) in the "that"-clause. (Of course there are mixed forms in which only some constituent is *de dicto*.) What we understand as listeners to the report is understood by the subject. In the *de re* reading the reporter claims that the belief has some objective *content*, however referred to. The sentence used in the report need not share its complete meaning with the sentence believed. It only shares its referential content. Any part of the sentence is open to extensional substitution. *De dicto* reports are essentially meta-representational. This need not be so for *de re* reports. The

person attributing the belief uses her own representational resources and need not even aim at claiming anything about the subject's representations.

Belief attributions expressed in natural language are meta-representational by quoting another sentence or using that other sentence in an embedded complement clause. Our sub-doxastic reasoning, however, will use such attributions as well. And the natural language reports have to have some conceptual content. The representation medium of these levels (the LoT) therefore has to have the means not only to build meta-representations in general, but to build meta-representations which contain items of the public language. Linguistic sharing of labour may use a structural description (say a quote of a form in one's linguistic community). In this case, supposing a successful hooking up to the target extension, at least the mediation between the new concept (a LoT-type) and the referent requires meta-linguistic representation.

Indirect contexts invite attributions of propositional attitudes which essentially point to *the way* the attributee represents a state of affairs. In such attributions one may meta-linguistically point to a speaker's idiolect, quoting an expression of the language to explain its usage by the attributee of the attitude ascription. The conceptual content of such an attribution thus contains a quotation or some other meta-linguistic device (like reference to phonetic or graphemic features [cf. already Kaplan 1969]). A word or part of a phrase is represented at a LoT level as a set of phonetic features, each of which has some LoT representation. Thus it is sufficient for quoting a natural language word or phrase to embed its representation into another LoT representation.

Such representations can serve *as labels* in file semantics, and play an explanatory role when dealing with indirect contexts. File semantics works with the idea that our knowledge is heavily compartmentalized. One compartment may contain my botanic knowledge about elm trees, another my knowledge about Cicero – and maybe another my knowledge about Tully. This solves a couple of problems: facts about the same object can be kept apart if they are filed in different places; keeping relevant facts from interacting may be an explanation of self-deception (cf. Davidson 1980). Merging files may be the use of informative identity statements.

Predicates and relations have to be categorized into intentional/indirect predicates and relations vs. direct/referential predicates and relations. With respect to the intentional ones we follow the principle that the second relate of an intentional expression is the expression used in the intentional attribution (or a logically equivalent expression). This corresponds to a representationalist understanding of indirect/intentional contexts. In case somebody thinks of something or believes something she stands in a relation to a representation. This representation (or a logically equivalent one) will be used in reporting the intentional state. Truth conditions of such reports are meta-representational (like: "Peter thinks of the unicorn" is true only if Peter stands in the relation thinking-of to the representation "the unicorn"). Our reports involving *non-*

referring singular terms in the scope of an intentional expression thus can be true or false (e.g. it may be true that Peter thinks of the unicorn, although there are no unicorns). In all intentional contexts one stands in relation to a representation, in some one stands additionally in relation to the referent of the representation. Intentional contexts are like quotations and need some specific rules of quantifying-in (e.g. requiring a referentiality assumption with respect to the expression related to – instead of an existence assumption with respect to some possible object).

[A theory of this type resembles a theory of same-saying (cf. Davidson 1968) or a quotation theory like (Capellen/LePore 2007).]

§11 A Representationalist Account of Fiction

Another challenge is statements about fiction, or truth in fiction (like “Sherlock Holmes lives in London”). From the perspective of a representationalist theory fictions should neither be treated as dealing with *possibilia* nor as some kind of abstract object, but as just *representing a story*. Fictions are narratives, which consist of representations, some of which are declarative sentences representing what the world is like according to the narrative. The narrative contains a lot of claims which are taken to be *true according to the story*. This involves deductive closure (e.g. “Sherlock Holmes does not live in Australia” is true according to Doyle’s fiction, although that very sentence never occurs in it). Deductive closure poses no difficulties here as one may understand the narrative as a set of sentences closed under some logic. The basis of deductive closure may also contain assumptions not present in the narrative itself, but taken for granted by the author (like gravitation holding in London the whole day) or supplied by theories we now know to be true and which do not contradict the narrative (like modern chemistry explaining Holmes’ little experiments). A sentence is true according to a story in case it can be deduced from it (i.e. from the set of its declarative sentences taken as true). Talking about fictional entities reduces to talk about representations.

Fiction is not false, since it does not aspire to be true, in contrast to error or lying. The felicity conditions of telling a story do not involve claiming the story to be true, and thus being responsible for some minimal warrant.

Reflections on Frege's Theory of Concepts

§1 Sentential Unity

Frege provides a theory of sentential unity. The distinction between concept and object he takes to be of crucial importance and to be one of his quintessential insights. The category distinction between concepts and objects explains sentential unity and why no further constituents can be added to a sentence at will.

Russell provides us with no theory of sentential unity. In fact his early (*The Principles of Mathematics*) theory of propositions seems to be unable to explain why no further constituents can be added and why we cannot simply get a proposition out of a collection of objects ('terms' in his then vocabulary).¹ Russellian propositions taken as abstract entities also seem unable to distinguish active and passive renderings of a sentence, as well as a complete reversal in the way θ -roles of a predicate are filled.

Do we need a theory of sentential unity?

Suppose we take (with Frege and Russell) truth to be basic (i.e. we do not try to define the concept 'truth'). Then we can separate statements from other sentences or word collections as those linguistic items that can be true or false: as the basic semantic unit as far as claims concerning reality are made. They are then a natural collection. We need not further explain what distinguishes them, supposedly not even what *makes* them true or false or what this quality consists in.

When we now look inside propositions or sentences we may do so without the purpose of explaining their unity. Their unity is explained, as far as its special status is concerned, by their ability to bear truth values. It can be taken as basic as soon as we start with taking truth as basic: there have to be units which behave the way statements behave. We recognize these units (i.e. statements) by their relationship to truth.

This *ratio cognoscendi* on the other hand may have a *ratio essendi* – what about statements makes them prone to be bearers of truth values? Frege's the-

¹ Reflections on Russell will easily suffer from mixing up the different views Russell developed between *The Principles of Mathematics* (1902) and *Introduction to Mathematical Philosophy* (1919) and *The Philosophy of Logical Atomism* (1918). The focus in this chapter is put on some common ground of Russell's theories of propositions and some systematic questions relating to them, so that the constraints of interpretative adequacy are weakened here.

ory explains the unity of them by the complementary features of their *constituents*. On a linguistic level Russell (at the time of the *Philosophy of Logical Atomism*) seems to follow suit at least in parts: propositional functions are defined as functions that have propositions as *symbolic units* as values. Propositional functions understood as schemata are working thus on the lines of Fregean concepts. The basic semantic unit (namely assertoric sentences or statements) has to be elucidated by analysis, and this analysis points to the crucial categorical distinction between the concept/propositional function and its argument(s).

Russell claims correctly then that we should understand general functions by first clarifying propositional functions, and not the other way round, as seems to be the case in Frege's classical papers "Funktion und Begriff" and "Begriff und Gegenstand". Frege's way of introducing concepts by first talking about function, may, however, only be a didactical device, as Frege can assume that his readers know functions in general and now have to realize the crucial role of concepts in logic.

§2 Frege on Sentential Unity

For a theory of sentential unity we need the claim that the constituent structure of a sentence needs two categorically distinct components: distinct in their syntax and their semantics. Frege's distinction between concepts and objects provides just that.

(SU) Theory of Sentential Unity

The unity of sentences involving first-order general terms (referring to first-order concepts) and singular terms referring to objects stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms referring to objects, which are saturated (as are their referents).

Frege's error lies in the move from the proper claim that some unsaturated expression needs to refer to an unsaturated entity, and that some saturated expression needs to refer to a saturating entity, to the improper generalization that no saturated terms can have unsaturated entities as their referents. This simple syntax/semantics-isomorphism is unwarranted; an unsaturated entity of one ontic level might be considered as saturated enough to serve as argument for another unsaturated entity of the next ontic level (like in a hierarchy of functions). Frege's theory, however, can be amended to this purpose; we might also want to elucidate further the image of 'unsaturatedness' in its linguistic and ontological dimensions [we address this below].

Frege on the one hand made use throughout of functions/concepts being arguments of higher order functions/concepts (as the logical systems of the *Begriffsschrift* and the *Grundgesetze der Arithmetik* are variants of Second Order Logic, including also relational expressions relating objects to concepts), but,

on the other hand, he sternly denied that 'the concept *horse*' denotes a concept. The latter denial of concepts as subject of a proposition or sentence was the main reason why Russell in *The Principles of Mathematics* did not follow Frege's theory of concepts. Russell substitutes 'propositional function' for Frege's 'concepts'. He denies Frege's categorical distinction between concept and objects and by this drops Frege's theory of sentential unity.

§3 Russellian Propositional Functions

Russellian propositional functions $\psi(\acute{e})$ and $\phi(\acute{e})$ are *equivalent* if propositions like $\psi(a)$ and $\phi(a)$ employing them have the same truth value for all arguments. $\psi(\acute{e})$ and $\phi(\acute{e})$ are *identical* if they have the same value for all their arguments, which means result in the same proposition respectively. [As propositional functions have propositions as values no intensionality is involved here.]

The sentences *expressing* (Russell 1902) or *being* (Russell 1918) the propositions have, given the recursive truth conditions, the same truth value even though they do not designate truth values. As propositions can be identical a logic capturing all logical and metaphysical truths must include a sign of propositional identity and respective axioms which mirror the axioms of identity for objects. Because propositions can be arguments of propositional functions, Type Theory introduces type distinctions which may forbid a proposition being the argument of its 'own' propositional function, or make the propositional function 'systematically ambiguous' as it applies once to objects and once to propositions (i.e. entities of different types, as objects as not truth bearers are distinguished from propositions).

Whereas the equivalence of $\psi(\acute{e})$ and $\phi(\acute{e})$ can be as easily ascertained as the identity of sets by verifying the truth conditions of respective sentences, the question of identity for propositions taken as abstract objects is much harder to answer. How can we determine whether such propositions are identical? One criterion whether to base the metaphysics of logic on propositional functions could be the complexity of their identity conditions and our knowledge of them.

A simple solution would be at hand if the constituent structure of the sentences expressing these propositions corresponded to the constituent structure of the propositions. This meant that language – at least logical form – guides our metaphysical picture. And we are back at a Fregean theory of sentential unity.

Russell's move from propositions as abstract entities *sui generis* to statements (assertive sentences) stems from problems of his early identity theory of truth and his refusal of sets/classes. In a proposition *sui generis* objects and relations/qualities are combined in some way. If they are combined in a straightforward sense of the relata really standing in the relation in question, there can only be true propositions as any proposition then corresponds – by identity –

to a fact. A theory of the semantic content of false sentences thus is impossible.

Another way of combination could be the way several elements of a set are joined in their set membership. In this case the relation is one element and the relata are others, without the relation relating the other elements. A corresponding ontology models propositions not as entities *sui generis* but as something like tuples (i.e. sets). As Russell wants to avoid sets – or at least admit them only as a manner of speaking about (predicative) propositional functions – he cannot model propositions this way. Therefore already in *Principia Mathematica* (1910) propositions as abstract entities *sui generis* give way to propositions as assertive sentences. Propositional functions have now to be understood as general terms/predicates.

This limits the cardinality of propositions and propositional functions to \aleph_0 – a problem for a foundation of mathematics, which we leave to the side here. Russell, being unaware of arithmetization, could not see this problem.

Russell's problems with sentential unity are aggravated by the absence of an account of universals/abstract entities. Acquaintance with universals need not mean by itself having the recognitional capacities to relate them to objects!

§4 Sentential Unity and Ontological Composition

One could arrive instead at the denotational theory that singular terms denote objects, general terms denote properties/qualities/concepts and assertoric sentences being evaluated as true or false only denote in case they are true. In this case they denote a fact. [One could otherwise claim that assertoric sentences do not denote at all, but in case of their truth correspond to a fact.] False sentences do not correspond to anything. The ensuing correspondence theory of truth then requires introducing facts either corresponding to true assertoric sentences of different logical form (e.g. disjunctive, universal etc.), or making assertoric sentences of non-atomic logical form in some truth conditional recursive fashion true.

Russell's fact ontology in *The Philosophy of Logical Atomism* advances at least the fundamental building blocks of such an account. Russell, however, also set forth at that time the *Tractarian* doctrine that sentences are not part of the world – with all its bizarre consequences.

Truth bearers are, properly speaking, eternal assertoric sentences, either asserted themselves or going back to an asserted indexical declarative sentence. Indirectly one may consider the indexical sentences themselves as bearers of truth. A true assertoric sentence denotes a fact (corresponding to the referential content of the thought expressed by the sentence).

Properties ('universals' in some sense of that term) are abstract entities in the sense of being a structure or structural component of an object (a chunk of reality), i.e. abstract in as much as they have no independent existence. We refer

to them as structures of reality by our concepts. A general term is unsaturated *because* its referent is an abstract entity dependent on an object. The singular term referring to this object taken as argument of the general term in question yields a true atomic sentence.

Such a theory would combine Frege's theory of concepts and objects with Russell's ontology of facts instead of having assertoric sentences denoting truth values.

We may picture facts by sentences, but facts do not become (true) sentences by this. Chunks of reality can be referred to by singular terms (and thus be considered as objects) or by true sentences (and thus be considered as facts). In this sense 'fact' and 'object' are ontological categories (i.e. categories employed within our ontological framework) covering the same pieces of reality. A chunk of reality has structure. Taken the structure into consideration we have a fact, otherwise an object. That we 'take into consideration' again doesn't mean that it otherwise isn't there. Object talk refers to objects. Fact talk refers to facts. Thus our ontology contains sentences (linguistic entities) and facts (chunks of reality) and no propositions *sui generis* in between or somewhere else.

§5 No Abstract Propositions

There are more difficulties with propositions as abstract entities.

If propositions are abstract entities, they are – at least once by the Theory of Descriptions reference to individuals has been eliminated – supposedly atemporal entities. They are, and they have always been there. How can we then reach out and contact them?

A theory of sentential unity may do some work here. Applying a general term to a singular term we also, in terms of meaning, relate the concept (or the relation) to the object (or the objects) in question – and *thus* grasp a proposition! Our account of our ability to create and understand sentences compositionally serves also as an account of our grasping of propositions. Our grasping is in this way also fine-grained enough to relate differently to propositions which, although logically equivalent, carve up reality in different ways.

Contrary to this the idea of everlasting propositions invites a theory of grasping in which the grasping of a proposition is a single undivided act, since one needs a theory of such acts of grasping anyway for universals which are part of such an ontology of compound abstract propositions. Sentential compositionality then becomes as *additional* issue, unrelated to grasping propositions.

A way out of this last dilemma consists in putting sentential unity and compositionality first in identifying propositions with sentences. Again a Fregean account carries the day.

In a Fregean theory of sentential unity applying a general term to a singular term corresponds as a linguistic activity to the semantic activity of relating the

referent of the general term to the referent of the singular term. If one takes the referents of general terms to be abstract entities (in the sense above) this semantic activity consists in recognizing or surmising the abstract entity as having the object as carrier or bearer (in case of general terms with one argument) or as recognizing or surmising the abstract entity as supervening on properties of the relata (in case of general terms with more than one argument). The ensuing unity of a sentence corresponds in case the sentence is true to the unity of the fact. Substituting an ontology of facts for Frege's ontology of truth values – and the accompanying peculiar thesis of statements denoting truth values – one can extend Frege's theory of sentential unity and increase its explanatory power, tying it more closely to a viable ontology.

Analysis of language (of sentential unity) and ontological analysis (of facts) proceed in unison, with linguistic analysis leading the way. Frege thereby appears once more as the true founding father of analytic philosophy, Russell's attempted ontological foundation turned out to be a blind alley.

§6 Kerry's Paradox

Frege considered his distinction between functions/concepts and objects as one of his main achievements. He employs it within his theory of sentential unity. General terms expressing concepts are unsaturated linguistic item which in combination with singular terms yield again saturated linguistic items (sentences). Filling the argument places of general terms distinguishes the built up of sentential unity from a mere list. This combination on the linguistic level has a corresponding combination on the level of reference. Functions (and concepts as functions from objects to truth values²) are unsaturated entities and their combination with objects yields saturated entities again. Frege thus asserts both a linguistic categorical dualism as well as an ontological categorical dualism. The entities in the two categories are, tautologically, categorically distinct – but can we say so?

Frege himself (in)famously claimed that

- (1) The concept *horse* is a concept.

is not true as the expression “the concept horse” is a singular term and thus designates an object! How can we speak then about concepts at all? Similarly

- (2) No concept is an object.

which looks like the expression of Frege's ontological categorical distinction cannot be true, or not even be properly built, as the supposed logical form of it

² In the following I speak mainly of concepts. The same story could be told about functions.

$$(3) \quad \forall x (\text{Concept}(x) \supset \neg \text{Object}(x))$$

treats concepts and objects as being of the same underlying (neutral) ontological category. Again concepts are assimilated to objects. If this is ontological and syntactic nonsense, how can Frege's ontology be expressed at all? Can the difference between concepts and objects only be *shown* in a properly regimented language like Second-Order Logic (SOL)? If that was the case, it would have wide philosophical repercussion concerning the possibility of meta-logic and meta-theory in general, as witnessed by Wittgenstein's *Tractatus*. Frege, at times at least, seems to settle with inexpressibility.³

§7 Metalinguistic Ascent?

In a corresponding meta-language one may speak about the expressions and their syntax. Truth conditions and semantic modeling show the categorical differences of the types. Nevertheless two problems stay with us:

- (i) Distinct statements are used to characterize the distinct types, which reveals again the type distinction (e.g. “If α is a general term ...”). Only if we have talked so far only about syntactic composition, everything sounds proper. This poses no problem as expressions *are* objects, and thus can be covered by a common category. We may even try to express truth conditions using schemata. This raises the issue what we understand when we understand schemata – supposedly a generalization about the involved types of entities.
- (ii) Attempting ontological type neutral meta-language talk we presuppose an understood type system of the meta-language. We may end up in a hierarchy of meta-language explanations of type distinctions.

In any case we have not expressed (2). Saying of two types A and B that they are distinct

$$(4) \quad \neg \exists x (x \in A \wedge x \in B)$$

again uses a domain of entities of neutral type. If x has a concept as value and concepts are essentially unsaturated, then $x \in A$ cannot be a thought or proposition *at all*.

Accordingly: If “Object()” in (3) was higher order (thus being able to take concepts as arguments) we might get a proper thought/proposition, but one which does no longer say what it was supposed to say, as its content becomes more or the less equivalent to

3 “Wenn es irgendetwas geben kann, was nicht Gegenstand ist, so kann man diese Tatsache nicht ohne Widerspruch aussagen.” Letter to Russell, June 24th 1902 (cf. for the quotes from the letters: Gabriel/Kambartel/Thiel 1980). Similar passages can be found in other letters to Russell. Frege explicitly resorts to semantic ascent: “Statt des ungenauen Ausdrucks ‘ ξ ist eine Funktion’ kann man sagen ‘()³⁺⁴ ist ein Funktionsname.” (Letter to Russell, June, 29th, 1902). Also cf. Ricketts 2010.

$$(5) \quad \forall x (\text{Concept}(x) \supset \text{Concept}(x))$$

Objects drop out of the statement. (2) is not expressed.

A completely universal quantification had to cover both concepts and objects. A general concept of 'entity' might serve this purpose. (3) and (4) would be all right then, quantifying over entities in general. Our problem reoccurs with the syntactic typing of the variables used then. We needed a neutral syntactic category as well. Concepts and objects would be values of variables of this type. Then expressions denoting values of variables of this type could be either general terms or singular terms. If we had the neutral (syntactic) category we could built sentences with combinations of concepts expressions and object expressions which are not intuitively well-built (say, as they switch the position of singular and general term). Given Frege's theory of the semantics behind sentential unity some combinations of 'entities' just don't suffice.

Should one even allow such combinations and postulate that they are all to be evaluated as false? As false they could still occur as consequence or premisses of inferences, which makes no sense. Even though it is true that non-well-formed expressions aren't true. This might be linked to a general principle:

$$(P) \quad \text{Syntactically non-well-formed expressions cannot be true.}$$

The Principle (P) is obviously correct, but does not solve the problem of expressibility either.

Pushing this observation from the meta-language into the language – a para-consistent language as we now are able to use a truth predicate or truth operators within a language – we get (given disquotation and the opposition of truth and falsity) from “False(“Peter(() is a horse)”)” to the syntactic nonsense “¬Peter(() is a horse)” again.

We seem to come closer to expressibility by semantic ascent. We might say, for instance:

$$(6) \quad \text{That which “() is red” denotes is a concept, and we cannot say of a concept that it is an object.}$$

We still cannot express (2) in its generality. In its intended syntactic reading the second conjunct of (6) asserts the impossibility of a well-formed sentence of some structure. We can *express* our inability to express an ontological truth, and we cannot express the ontological truth itself. We cannot even express why we cannot express this ontological truths, it seems.

Our next attempt in ascent might be:

$$(7) \quad \forall x (\text{ConceptWord}(x) \wedge \text{Welldefined}(x) \supset \exists F (\text{Denotes}(x,F)))$$

Now, we need to introduce *two concepts of denoting* as the one takes a second-order argument and the other a first-order argument! This may seem unfortunate as denoting should be the same relation for all expressions, but it may still be so, our theory of denoting may explain as much. Only our relations of denoting have for categorical reasons to be as many as there are categories.

The truth condition of 'saying of ()' may then state that one may *say* a general term α of some entity x which is denoted by β iff $\alpha(\beta)$ is syntactically correct, talking about linguistic entities only.

Denoting should get us to the referent of the linguistic item, thus “what '() is red' denotes” should be an expression getting us at the concept expressed (the *referent* of “() is red”). We may, following Frege in some of his remarks (cf. Frege 1971: 31; a never published manuscript), say even:

(8) The book is what '() is red' denotes.

This seems a bit cheating, however, as “what '() is red' denotes” still is a singular term, and, therefore, should, by Frege's own original argument, be denied to denote a concept.

All the proper work done in (8) is done by “() is ()*”.

(9) My pencil what '() is red' denotes.

is no sentence. [Dummett (1981: 213-14) follows Frege here and papers over the role of “is”, which cannot be left out of the sentence.]

In this mood of linguistic ascent we may further proceed to

(10) $\forall F$ (“Object()” cannot be said of F)

and correspondingly for objects and “Concept()”. No concepts can be said to be an object as that combination cannot be syntactically correct: “Object()” should take *first-order* arguments.

We still cannot say *why* some combinations of signs are semantically incorrect. We can stipulate that they are syntactically incorrect, but such stipulations ultimately depend on a semantic picture (like Frege's picture of concepts as unsaturated).

Frege's demand for determinacy of concepts

(DET) $\forall F \forall x (F(x) \vee \neg F(x))$

is a formula satisfied by concepts only, and thus might be seen as defining 'concept' and the corresponding category, the open formula (without the universal quantifier on concepts) being true only of concepts, which, however, is enforced by the syntax of the argument in the first place.

Given a theory of extensions or sets we may define in our meta-language

$B = \{x \mid x \text{ is denoted by a general term}\}$

$G = \{y \mid y \text{ is denoted by a singular term}\}$

and as extensions/sets are first-order entities we can now say

(11) $B \cap G = \emptyset$

A set theory like ZFC allows us to say as much, but ZFC itself introduces a categorical distinction concerning the status of the relation $[\in]$ which has no extension in ZFC. ZFC itself works with an ontological categorical distinction between its sets and non-set collections (like the universe V itself). The debate

would change only in details if we considered this distinction. ZFC gets rid of concepts in favour of sets, but contains its own inexpressible ontological duality, even if it is only presupposed. Additionally we may nonetheless need a theory of concepts as part of our theory of sentential unity.

The paradox of the concept *horse* points to a fundamental dilemma: Either we do not distinguish semantic types (i.e. types in the reference of expressions) and cannot explain sentential unity, or *we do so* being able to explain sentential unity, but have traded in the problems of expressibility. The difficulties of the mentioned hidden type distinctions in ZFC point to a similar dilemma.

Mixed-level general terms have arguments of different ontic levels (e.g. concepts and objects).

Frege allows for mixed-level general terms (the most well-known being “() is the value range of ()*”). And even if Frege had not done so, there is no principled objection against mixed-level general terms. If mixed-level general terms are allowed we may introduce the predicate “categorically distinct” and say:

$$(12) \quad \forall x \forall F (x \text{ is categorically distinct from } F)$$

To introduce the predicate we have to stipulate the first argument to be first-order and the second argument to be second-order. This can be said as we talk *in this stipulation* about linguistic items. What we have not done – still – is to explain what “categorically distinct” means. Once we move towards an explanation we enter ontology again and face the problem of (2).

Using a device like (12) we can justify and establish our symbolism (say in SOL). A statement like (12) repeats what is shown by the use of different types of variables. Note, however, that (12) does not talk about linguistic items, but about concepts and objects themselves! We may achieve more than what simple semantic ascent can give us. Ascent elucidates our usage of different variables and expressions, whereas (12) reaches the ontological level. Therefore we also not just deal with a 'saying/showing' type of elucidation here. And the step to semantic ascent provides no necessity here to drop ontology in favour of philosophy of language. What is left open is an ontological thesis corresponding to our linguistic thesis. (2) cannot be said – but should we bother? We can say that concepts and objects are categorically distinct and this concept of categorical distinction is embedded in our general theory of language and ontology. Some concepts have to be basic, so why not 'categorically distinct'?

§8 Inexpressibility Limited

So, how far does inexpressibility go? To express (2) may be an expectation stemming from our ordinary distinction of categories, which should not be taken over lightly to fundamental matters. So – maybe – (12) is all we get, and all we need. On second sight a problem of expressibility reoccurs. (12) allows

us do distinguish objects and functions/concepts *of some definite order*: the second argument of the expression “() is categorical distinct from ()*” has to be of some specific order (say allowing to apply the expression to functions/concepts of objects), and thus does not allow its application to functions/concepts of the next order! So we may say, for instance, that concepts of objects are categorically distinct from these objects, but then what about second-order concepts? We seem to need another expression “() is categorically distinct from ()*” with the arguments being second- and first-order functions/concepts – and then another one...

We land in a hierarchy of such concepts and statements of categorical distinction, corresponding to Frege's hierarchy of functions/concepts. Even with a basic predicate and concept of categorical distinction we cannot express *in general* (i.e. across the whole hierarchy) that concepts are categorically distinct from objects (and from each other according to their order). Strict distinctions of type and order present the dilemma – besetting also Russell's type theory or other hierarchies like Tarski's semantic hierarchy – that either some features of the structure of the hierarchy are inexpressible or in our attempt to express them in conveying the hierarchy we land ourselves in performative inconsistency (doing what our theory says cannot be done).

For Frege the ensuing regress of ever more levels of “() is categorically distinct from ()*” might be seen as virtuous instead of vicious, as we may resort to such statements when needed, and above the third level there are no crucial applications of such statements, given that Second Order Logic is all we need. A shortcoming of such a proceeding is that we either introduce the expressions of our formal language (namely the statements of category distinction) piecemeal or by a generic statement or schema about expression of the general form “() is categorically distinct from ()*”, which cannot be part of our theory itself again. We need ascent to a richer meta-language then, which gets us into conflict at least with the conception of logic being completely universal. As Frege sometimes employs meta-logical arguments, he might have accepted this form of ascent, and it certainly is common practice today.

Assuming Frege's approach struggles with problems of expressibility, can we keep Frege's account of sentential unity and allow concepts to be referred to? Once we allow for ascent to a hierarchy of concepts and a meta-language, there seems to be a solution. We have to combine Frege's thesis on sentential unity with the claim that concepts can be subjects of sentences and can be referred to by (special) singular terms. For a theory of sentential unity we need the claim that in the constituent structure of a sentence we need two categorically distinct components, distinct in their syntax and semantics. Frege's distinction between concepts and objects provides just that. As mentioned at the very beginning: Frege's error lies in the move from the proper claim that some unsaturated expressions need to refer to unsaturated entities, and that some saturated expressions need to refer to saturating entities, to the improper gen-

eralization that no saturated terms can have unsaturated entities as their referents. The syntactic/semantic-isomorphy on unsaturatedness is unwarranted.

The account then may be this:

(SU) Theory of Sentential Unity

- (i) The unity of sentences involving first-order general terms (referring to first-order concepts) and singular terms referring to objects stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms referring to objects, which are saturated (as are their referents).
- (ii) Singular terms of the form “the concept ___” refer to concepts.
- (iii) Singular terms of the form “the concept ___” with the concept filling the slot being of order n saturate general terms referring to concepts of the next order, $n+1$.

This account differs from Frege's in introducing a hierarchy of singular terms corresponding to the hierarchy of concepts, but keeps the essential account of sentential unity. Frege himself in employing Second Order Logic allows for concepts being the arguments of higher-order concepts so that their own unsaturatedness does not make a sentence/thought about them unsaturated, once the higher order concept has been saturated by them. Clauses (ii) and (iii) in (SU) make generic claims about concepts *in general* (i.e. across the whole hierarchy of concepts), and thus have to be made in a meta-language, which ultimately faces the same problems itself, giving rise to another meta-language – and so forth, as it is with these hierarchies of meta-languages. This undermines the idea of logic as truly universal, but cannot be held against a Fregean account by those who employ similar hierarchies in their own approaches.

If you are dissatisfied with the virtuous or vicious regress to meta-languages, the ultimate solution could be to drop the distinctions between orders of concepts and use one level of concepts only, i.e. forsake any hierarchy in the objects or in the concepts. This would allow for a simplified theory of sentential unity.

(SSU) Simplified Theory of Sentential Unity

- (i) The unity of sentences involving general terms (referring to concepts) and singular terms stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms (respectively the concepts being saturated by the referents of the singular terms).
- (ii) Singular terms of the form “the concept ___” refer to concepts.
- (iii) Each sentence (thought) has exactly one constitutive general term (concept).

- (iv) The saturation of the thought depends only on the saturation of the constitutive concept (i.e. a concept referred to by a singular term in argument position can stay unsaturated).

This theory differs from Frege's both in having singular terms refer to concepts – as does (SU) already – and in *dropping the hierarchy* of concepts. Concepts now can be applied to themselves, and “() is categorically distinct from ()*” can be applied to state that the concept of categorical distinction is distinct from any object. We may even allow a common domain then, containing both objects and concepts, with quantifiers running over both of them, which – finally – allows us to express (2). [This resembles Russell's early use of the neutral category 'term' in his *Principles of Mathematics*.]

The downside of this vanishing of hierarchies and self-application of concepts are, of course, the ensuing contradictions. A general category of concepts and corresponding general terms immediately yields the “heterological”-paradox. The distinction between objects and concepts, being expressed thus, then requires a move to paraconsistency – i.e. to an approach supposedly anathema to Frege and his followers.

What have we seen? – Frege's ontological distinction can be expressed if we commit us either to a hierarchy of basic concepts covering categorical distinction or to a paraconsistent non-hierarchical universal language. Both approaches may amend Frege's theory in allowing for some singular terms referring to concepts, as this does not endanger Frege's theory of sentential unity.

§9 Concepts and Logicism

Frege's theory of concepts lays not only the foundations for his theory of sentential unity, but also lays the foundations for his logicism by introducing ‘value ranges’ (extensions of concepts) as the fundamental entities for mathematics.

The following paragraphs reconsider the ontological and logical issues around Frege's *Basic Law (V)*. They focus less on Russell's Paradox, as most treatments of Frege's *Grundgesetze der Arithmetik (GGA)*⁴ do, but rather on the relation between Frege's *Basic Law (V)* and *Cantor's Theorem (CT)*. So for the most part the inconsistency of Naïve Comprehension (in the context of standard Second Order Logic) will not concern us, but rather the ontological issues central to the conflict between (BLV) and (CT). These ontological issues are interesting in their own right. And if and only if in case ontological considerations make a strong case for something like (BLV) we have to trouble us with inconsistency and paraconsistency. These ontological issues also lead to a renewed methodological reflection what to assume or recognize as an axiom.

4 I will use “GGA” as abbreviation both for the book and the logical system developed in the book and disambiguate only if a context makes this necessary.

§10 Value ranges and extensions

Frege's *Basic Law (V)*⁵

$$(BLV) \quad (\forall F, G)(\acute{e}F(e) = \acute{e}G(e) \equiv (\forall x)(F(x) \equiv G(x)))$$

states that for any two concepts⁶ it is true that their respective *value ranges* are identical if and only if their applications to any objects are equivalent. As is well known value ranges are the new ingredient in GGA that supplements the system of the *Begriffsschrift*. The logic of (the book) *Begriffsschrift* is SOL without comprehension principles. With GGA Frege takes 'value range' as a basic concept, which can be illustrated by considering the graph of a function. Value ranges thus resemble sets of ordered pairs, but as a basic concept 'value range' is undefined and could thus itself be used in introducing ordered pairs. Frege also often speaks of the 'extension' of a concept, as if "extension" meant almost the same as "value range". Extensions understood as sets can be gained from value ranges by considering only those objects that are mapped under a concept to the truth value TRUE (i.e. by considering a projection on the corresponding value range)⁷. Thus, although Frege does not use set theory in the standard sense of ZFC and related systems, but refers only to his value range objects, we may talk about extensions as sets and (BLV) as stating an identity condition on extensions (and thus sets).

Frege defines a function ("x ↦ éF(e)") of an object and a value range that outputs on the input of an object and a value range the value the object is mapped to under that value range. So if we consider the function 'father-of ()' and George the function in question outputs, say, Lloyd. If we consider concepts,

5 Standard symbols are used here instead of Frege's *Begriffsschrift* notation. In contrast to Frege identity between objects ("a = b") is distinguished from equivalence between statements ("F(a) ≡ G(a)"). The debate about statements as names for truth values does not affect the issues discussed here.

6 (BLV) holds in GGA also for any *functions* whatsoever. We consider only *concepts* (i.e. functions from objects to truth values). For our purposes here we do not consider relations either. Relations could be reduced to concepts anyway (e.g. simply by working with concepts like "() being the father of George" instead of working with the relation "() being the father of ()*" if one does not quantify in the argument places of the relation; in that case relations cannot be reduced to concepts as some sentences with relation expressions are only satisfiable in a non-finite model (as Frege himself shows with respect to the Dedekind/Peano-Axioms), whereas satisfiable sentences with monadic predicates are satisfiable in a finite domain.

7 "TRUE" is used here as referring to the truth value TRUE, a basic object. Frege assumes truth values as basic objects. For mostly technical reasons he identifies them with some extensions (ultimately their own singletons) in GGA. We follow Frege's commitment to truth values here as nothing in the argument *here* depends on the issue whether asserted eternal sentences refer to truth values or just are evaluated with respect to their truth (and refer to something else or do not refer at all). The whole debate could be restated with eternal sentences referring to facts or the world. I prefer understanding true sentences as referring to facts, as outlined in Chapter 1.

say ‘is-prime()’, and do the same with ‘is-prime()’ and the object 2 we get TRUE. When we only consider concepts the function in question between a value range/an extension of a concept and an object outputs TRUE if and only if the object is mapped by the concept to TRUE, i.e. if and only if the concept applies to the object. Thus, concerning concepts and talking of extensions as sets the function defined by Frege corresponds to set membership (expressed by “ \in ”) and given (BLV) the following abstraction principle holds

$$(NCF1) \quad (\forall F)(\forall x)(x \in \acute{e}F(e) \equiv F(x))$$

This resembles λ -conversion and is similar to the common expression of *Naïve Comprehension*, as we develop more closely in §11. Although Frege uses a relation wider in definition than elementhood – as he used with ‘value range’ a concept wider than ‘extension’ – elementhood is present in GGA, and thus we can freely use “ \in ” and set abstracts (like $\{x \mid F(x)\}$) without doing anything that could not be done in GGA.

§11 Abstraction and Comprehension in *Basic Law (V)*

With (BLV) GGA adds an *abstraction principle* to the system of the book *Begriffsschrift* (i.e. basic SOL). We abstract from the different ways we come to or compute some value range by using any one of the group of concepts that share a value range to identify that value range. Concepts taken thus are concepts in extension (extensional concepts). [Frege has no use for intensional entities (and thus for concepts in intension) in GGA.]

From (BLV) we get by substituting twice with the same concept

$$(13) \quad \acute{e}F(e) = \acute{e}F(e) \equiv (\forall x)(F(x) \equiv F(x))$$

as the right hand side, just as the left hand side, is a logical truth we can detach to

$$(14) \quad \acute{e}F(e) = \acute{e}F(e)$$

and then, as extensions/value ranges are first order objects, existentially generalize to

$$(15) \quad (\exists x)(x = \acute{e}F(e))$$

and then again generalize on the second order constant to get *Naïve Comprehension* as an existence claim

$$(NCF12) \quad (\forall F)(\exists x)(x = \acute{e}F(e))$$

The numbers in the name “NCF12” indicate that the comprehending object is first-order and the statement quantifies over concepts (i.e. is second-order). In more modern notation and using the presence of elementhood, as expressed in (NCF1), we can write more generally

$$(NC12) \quad (\forall F)(\exists x)(\forall y)(y \in x \equiv F(y)) \quad [“x” \text{ not free in “}F(\)”]^8$$

8 Unless mentioned we understand this requirement to be expressed and fulfilled.

the first order, schematic version of which is

$$(NC1) \quad (\exists x)(\forall y)(y \in x \equiv \varphi(y)) \quad [“x” \text{ not free in } \varphi]$$

where φ is any open formula ('propositional function') of the language. With the help of set abstracts we can write also

$$(NCF12') \quad (\forall F)(\exists x)(x = \{y \mid F(y)\})$$

There is also a second-order version of comprehension

$$(NC22) \quad (\exists X)(\forall y)(X(y) \equiv \varphi(y)) \quad [“X” \text{ not free in } \varphi]$$

We will refer to these comprehension principles, if no distinction between these versions is relevant, just as “(NC)”. We should note, however, that

$$(MT1) \quad (BLV) \vdash_{GGA} (NC12)$$

(BLV) leads to first-order entities (namely objects) comprehending first-order entities which fall under a concept. Frege's domain of objects is unstratified [as discussed below] so all and only objects are first-order entities. Frege distinguishes functions of objects from higher-order functions.

GGA without (BLV) allows to derive (NC22)

$$(MT2) \quad \vdash_{GGA \setminus \{(BLV)\}} (NC22)$$

because Frege has a substitution rule that treats propositional functions like concepts: they can be interchanged in theorems with unbound second-order variables and one can instantiate second-order variables to propositional functions. Starting with

$$(16) \quad (\forall F)(\forall y)(F(y) \equiv F(y))$$

we get by *Basic Law Iib* of GGA (which is Universal Instantiation for concepts):

$$(17) \quad (\forall y)(\varphi(y) \equiv \varphi(y))$$

and then again by second-order Existential Generalization:

$$(18) \quad (\exists X)(\forall y)(X(y) \equiv \varphi(y))$$

In a Fregean context, using (BLV), (NC) expresses the idea that for every concept there is an extension (of that concept).

As often retold, (NC12) leads to *Russell's Paradox*, since it assures us that (i) the set $\{x \mid x \notin x\}$ has to exist as the concept ‘ $x \notin x$ ’ exists, and (ii) that this set is a first order object itself, able to fall under that very concept, which leads to the contradiction (of it being a member of itself and not being a member of itself). As also often retold Russell discovered his paradox by working through the proof Cantor gave of (CT); in effect Russell's argument is only a special case of Cantor's more general argument. And Russell told Frege about this in a letter [more of which later].

§12 Comprehension and *Cantor's Theorem*

(BLV) ensures that two extensionally distinct concepts have distinct extensions. (NC12) then expresses a correspondence between the domain of quantification of the upper case letters and a sub-domain (proper or not) of the domain of quantification of the lower case letters; the version with the order of quantification switched (i.e. $(\forall x)(\exists F)(\forall y)(y \in x \equiv F(y))$) would express that every object is the extension of some concept, a principle which Frege actually supports in GGA, which is concerned only with truth values, which Frege identifies with some extensions (namely their own singletons), and mathematical objects like numbers, which Frege constructs as extensions, but $(\forall x)(\exists F)(\forall y)(y \in x \equiv F(y))$ cannot be accepted for a general application of logic. Even for extensional concepts – intensional concepts could only introduce more distinctions – there have to be as least as many objects as there are higher order entities. Since any two distinguished extensional concepts are distinguished in their application to some object at least, so that they have different extensions by (BLV).

This stands in direct contradiction to Cantor's Theorem, that there have to be more sets of order n corresponding to the propositional functions (concepts) on objects of order $n-1$ than objects of order $n-1$; in its general form, quantifying over sets x :

$$(CT) (\forall x)(|\{0,1\}^x| > |x|)$$

The cardinality of the (set of) functions from some set x to $\{0,1\}$, which may be thought of as concepts mapping objects from x to TRUE or FALSE, is strictly larger than the cardinality of the set x in question. A more familiar expression talks instead of these functions of the subsets of x they generate, so that we can write – using the concept of 'powerset':

$$(CT') (\forall x)(|\wp(x)| > |x|)$$

Looking at (CT) and its talk of function we can say: There are *more* propositional functions/concepts in extension (with respect to the elements of x) than there are objects (in x). A rephrasing directly relevant and in contradiction to (NC12), as a collection of objects (namely value ranges), according to (NC12), is at least as numerous as the collection of concepts that they fall under at the same time.

We can thus immediately recognize that combining (NC12) with a system including set theory enough to yield (CT) *has* to be inconsistent.

§13 Frege and *Cantor's Theorem*

Of course, Frege was surprised by *Russell's Paradox* and he did not see the problems engendered for referentiality that come with (BLV) [see §14]. He explicitly, however, derives his form of (NCF1), and thus would have supported (NC12). One of his tactical manoeuvres consists in exploiting (BLV)

and (NC12) to substitute talk about extensions (i.e. first order objects) for talk about concepts. He can avoid higher order functions of degree more than second order in GGA by substituting for a concept as argument of a higher order function the extension of that concept in a closely related first order function (GGA §§35, 37). Thus he doubly requires a correspondence both between concepts and their extensions as well as between second order and first order functions, violating (CT) twice over.

Why did Frege not see the supposedly obvious contradiction between (CT) and (NC12)?

Frege knew Cantor's work in general, but could he have overlooked or not known (CT)? Cantor developed (CT) stepwise. Around 1873 he discovered the version dealing with the case of real numbers and published it in 1874 and 1878 (the famous "Beitrag zur Mannigfaltigkeitslehre"), the powers of sets are developed in 1883 (in the even more famous *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*), and the diagonalization argument, which later inspired Russell to his paradox, was presented publicly and published in 1891 ("Über eine elementare Frage der Mannigfaltigkeitslehre"), the explicit argument in terms of the powerset he pointed out to Dedekind in a letter only (cf. Dauben 1979). Zermelo in 1908 called (CT) "Cantor's Theorem".

Frege mentions the *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* in his own *Grundlagen der Arithmetik* (GLA) with respect to Cantor's theory of the infinite cardinalities, but they then quarrel only in an exchange about Cantor's 1885 review of GLA about the question whether Cantor's cardinalities ('Mächtigkeiten') are the same as Frege's ('Anzahlen'). Cantor in fact warned to take extensions as a basis of a theory of numbers, but the idea that not all collections are sets, that some of them are too large, came to Cantor only around 1890, from which time on he knew about the paradoxes of the universal set and – supposedly – the *Burali-Forti Paradox*, but he did not publish these insights (cf. Hallet 1984: 126-28, 165-75).

In the second volume of GGA Cantor is targeted for his formal imprecision and supposed confusion between sign and abstract object, and is supported again concerning the existence of the actual infinite. Cantor's major work in set theory Frege does not discuss in any detail, and so one may think that he just missed the discovery of *Cantor's Theorem*, however difficult to believe this sounds. Frege showed no interest in general set theory beyond the use of extensions at the foundation of arithmetic and reserved judgement on the role larger ordinals may play. Frege had to know *Cantor's Theorem* and its standard proof before the publication of the second volume of GGA, if only because of Bertrand Russell. In the second volume Frege in fact mentions the different cardinalities of the collection of all finite numbers and the collection of all value ranges comprehending finite numbers (GGA §164), but he does not refer to Cantor or his arguments in this connection.

Russell in his correspondence with Frege explains his discovery of his paradox (in a letter from June 24th 1902) by his study of Cantor's proof that there is no largest cardinal number, which he supposes Frege knew. Given Cantor's diagonal proof that there can exist no correspondence between a set x and its powerset, *Russell's Paradox* just results as the special case of considering the universal set and the identity function as correspondence. In another letter to Frege (July 24th 1902) Russell explicitly says that one can easily prove that there is *no correspondence* between all objects and all functions and in a letter from September 29th 1902 even outlines his formal version of (CT) mentioning again Cantor's claim about powersets. Frege first doesn't reply to the allusions to Cantor and comes up (in a letter from August 3rd 1902) with the astonishing remark that he considers the proof that there can be no correspondence between all objects and all functions as 'questionable' ('bedenklich'). He argues that the very idea of correspondence and uniqueness presupposes the notion of identity, and identity is a first order concept! (Remember that GGA and SOL use "=" only for objects and concepts are compared only with respect to co-extensionality by means of universal quantified equivalences using "≡".)

This is astonishing in two respects: Firstly, it seems that Frege flatly denies (CT) or at least Cantor's proof! Secondly: even if concepts do not enter in identity statements, Frege's formal system allows for mixed relations (having as one argument an object and as another argument a concept) and Frege uses such mixed relations himself (the most crucial being, of course " $\hat{}$ " which relates a concept to its extension). In terms of such mixed relation a correspondence between all objects and all functions can be considered without obviously presupposing identity. The 'uniqueness' presupposed in a correspondence (in the sense of equinumerosity) is functionality, which makes use of identity between objects (like in Frege's own definitions in GGA §§37-40):

$$(19) \text{ Funk}(F) \stackrel{\text{def}}{=} (\forall x,y,z)(F(x,y) \wedge F(x,z) \supset y = z)$$

Defining a correspondence strictly in that fashion it seems one is forced to apply identity to concepts⁹

$$(20^*) \text{ Funk}(M_{xy}) \stackrel{\text{def}}{=} (\forall F,G,H)(M_{xy}(F,G) \wedge M_{xy}(F,H) \supset G = H)$$

This reasoning seems to lay behind Frege's remark to Russell. Obviously, however, there is a simple solution to that problem: Let R be a mixed relation having objects as first argument and concepts as second argument, we then define:

9 We use " M_{xy} " to indicate that M is a second order relation, the common practice of using "F" etc. just like ordinary arguments (i.e. not in their Fregean form of " $F(\)$ ") hides their possession of argument places, which for Frege constitutes the crucial difference between concepts and objects. Writing just " $M(F)$ " at least depends on a common understanding that we cannot have also " $M(a)$ ".

- (21) Correspondence(R_y) $\stackrel{\text{def}}{=} \begin{aligned} & \text{(i) } (\forall x, G, H)(R_y(x, G) \wedge R_y(x, H) \supset (\forall z)(G(z) \equiv H(z))) \\ & \text{(ii) } \text{Funk}(R_y^{-1}) \end{aligned}$

(21) states the crucial concept of correspondence within Frege's own language: as Frege thinks only of concepts in extension two concepts equivalent in their application to objects have to be identified, captured in the first condition on a correspondence in (21), and a correspondence is functionally reversible, captured in the second condition in (21).

Starting with (21) we can proceed on the lines of Cantor's proof. We can also run an argument resembling Cantor's and Russell's argument with respect to Frege's correspondence $\dot{\prime}$ between concepts and their extensions: we start with

$$(22) \quad r \stackrel{\text{def}}{=} \acute{e}((F)(e = \acute{a}F(a) \supset e \notin e)$$

and re-run Russell's argument with respect to: $r \notin r$. Frege's remark against Cantor's argument thus seems widely off the mark.

Frege having not seen the incompatibility between (NC12) and (CT) should not stop us, however, from further investigating the issues involved in that conflict. Still we put aside the inconsistency of (NC12) itself (in combination with standard SOL or FOL) for the moment and focus on the ontological questions involved.

§14 Determinacy of concepts and reference and (BLV)

As the universe of GGA is flat and not stratified we can neither assume that some objects are being created later than others nor that they are acceptable by being structurally proper placed on a level of stratification which presupposes another level. In the cumulative hierarchy associated with ZFC the sets in higher ranks are not created later than those on lower ranks either, but they are structurally properly placed as all their members are on ranks below them. Thus taking the iterative hierarchy to be in place all at once – not being the result of some mystical temporal process of construction – does not exclude structural dependencies that resemble dependencies in a process of generation. The introduction of value ranges/extensions corresponding to all propositional functions/concepts as expressed in (NC12) thus poses a difficulty given Frege's principle that any concept has to be defined for all objects (as otherwise *tertium non datur* will not hold and his principle excluding stepwise definitions can be violated). For later reference let us re-call this principle:

$$\text{(DET)} \quad (\forall F)(\forall x)(F(x) \vee \neg F(x))$$

As value ranges are introduced as objects concepts have to be applied to them as well, to all of them. If we now introduce an extension $\acute{e}F(e)$ all concepts have to be applied to it, including F itself. If again we think of the relation of concepts to their extensions not as a temporal proceeding we can suppose that

with a concept we also already have the determination of the application of that concept to its own extension given. But given that we can proceed further and define propositional functions involving parameters for extensions the application of (BLV) as determining the identity of extensions runs into trouble.¹⁰

If we define:

$$(23) \quad G(_) \stackrel{\text{def}}{=} (_) = \acute{e}F(e)$$

we define the concept of something being the extension of concept F. G has an extension, and we can ask whether it is identical to the extension of any other concept. If we consider F we have to determine

$$(24) \quad \acute{e}G(e) = \acute{e}F(e)$$

But when we now look at the following instance of (BLV)

$$(25) \quad \acute{e}G(e) = \acute{e}F(e) \equiv (\forall x)(G(x) \equiv F(x))$$

we will find ourselves in the situation to determine (as an instance of the right hand side)

$$(26) \quad G(\acute{e}G(e)) \equiv F(\acute{e}G(e))$$

which requires to evaluate

$$(27) \quad G(\acute{e}G(e))$$

which brings us by the definition of “G()” full circle to

$$(24) \quad \acute{e}G(e) = \acute{e}F(e)$$

One might also proceed as follows:

By means of (NCF1) and (26) we get

$$(28) \quad \acute{e}G(e) \in \acute{e}G(e) \equiv \acute{e}G(e) \in \acute{e}F(e)$$

and now using (23) we arrive at

$$(29) \quad \acute{e}G(e) = \acute{e}F(e) \equiv \acute{e}G(e) \in \acute{e}F(e)$$

so that the evaluation of (24) asks us to evaluate

$$(30) \quad \acute{e}G(e) \in \acute{e}F(e)$$

With the circularity in the first argument we seem to meet an object the identity of which is indeterminable. The second argument avoids this, but only because it, with (NCF1), makes essential use of Naïve Comprehension, which

10 Cf. (von Kutschera 1989: 127-29). Frege's fixing of (BLV) in the appendix to the second volume of GGA tries to solve this very problem by exempting the extensions themselves from the verification of conceptual equivalence. The way Frege puts this implies unfortunately that there are only two objects. It makes extension as well to second-class objects (i.e. foregoing the usual logical treatment of objects).

otherwise, apart from Frege's proof of (HP)¹¹, can be avoided. And (30) still poses a foundational conundrum with respect to its evaluation: to evaluate (30) we have to know what object $\acute{e}G(e)$ is, but to know this we have, by (24), to know what object $\acute{e}F(e)$ is, and we can know this only, if we have evaluated (30), for which we have to know what object $\acute{e}G(e)$ is.

This in itself is not a contradiction, but it renders, for a start, Frege's attempted proof that every name in GGA has a proper reference useless. If every name (including sentences for Frege) had a proper reference, then GGA would be consistent (arguing on the stepwise increase in the complexity of names). Frege's attempted proof (GGA §31) fails as we cannot determine or prove proper referentiality (even if it obtained).

Frege's preference for (BLV) over other abstraction principles like (HP) and his argument around the 'Julius Caesar problem' are based on identity determination by (BLV) being generally feasible. As this is now falsified we have another reason (in addition to the inconsistency stemming from (BLV)) to switch to a reformed system like so-called "Frege Arithmetic" (i.e. GGA without (BLV), but with (HP) as additional axiom). Frege Arithmetic (FA) is consistent and can derive Peano Arithmetic (cf. Heck 2011).

FA still has a flat universe of objects, but does not entail (NC12) – as otherwise it couldn't be consistent. We can thus no longer be sure that every concept has an extension. More strongly put: we now know, since (NC12) *cannot* be true, that there *are* concepts without extensions. Can we accept this? [More on this later.]

A flat universe in itself is consistent, but only as long as we avoid new comprehension schemas. Once we introduce even limited comprehension like in the Axiom (Schema) of Separation, as is done in ZFC, we run into further difficulties like now being able to prove the inexistence of the universal set and so the inexistence of absolute complements¹², both of which are crucial ingredients in GGA (and FA).

§15 Where to put the blame?

As the concept of subset is elementarily tied to the concept of set, the Power-set Axiom (of ZFC) seems beyond reproach from an informal perspective. Simple observations within finite set theory corroborate that there are more subsets to a non empty set than one for each element, thus the set of the subsets of a set exceeds that set in cardinality (in finite set theory at least). These

11 (HP) states that the cardinalities of two concepts coincide if and only if there is a correspondence between the extensions of the two concepts (if and only if they are equinumerous). On the centrality of (HP) see below.

12 The proof as outlined in many set theory *text* books being again a *variation* on *Russell's Paradox*.

observations are not sufficient for infinite sets (as enumerations of \mathbb{Q} show), and here (CT) comes in.

Proving (CT) works by diagonalization or with an indirect argument. This argument need not be valid in non-standard logics, but is in SOL and in GGA. (NC) might so – presumably – be combinable with a non-standard logic which does not allow deriving (CT).

One idea might be to avoid the additional content of (NC) postulating a 1st and 2nd- order correspondence (additional to the idea of every concept having an extension) and *only* claim that there is an extension to any concept. These extensions then cannot be 1st-order objects given (CT) and SOL. Where to put them? Full SOL contains the Comprehension Schema (cf. for instance Shapiro 1991: 66)

$$(NC22) \quad (\exists X)(\forall y)(X(y) \equiv \varphi(y)) \quad [“X” \text{ not free in } \varphi]$$

This asserts that there exists a second-order entity the application of which corresponds to the application of some propositional function of the full language. (NC22) ensures us that all propositional functions can be comprehended into a second-order entity, and thus usually correspond in the semantics to subsets of the first-order domain. (NC1) ensures us that all predicates/propositional functions of the language are comprehended into a first-order entity. As φ is understood to be a propositional function for first-order arguments we revert, for now following Frege's equation of concepts and propositional functions, to

$$(NC22+) \quad (\forall F)(\exists X)(\forall y)(X(y) \equiv F(y))$$

If we now understand “ $X(y)$ ” as functional application, then (NC22+) becomes almost vacuous: “ $X()$ ” is just another – or even the same – first order propositional function/concept as “ $F()$ ”. If we understand “ $X(y)$ ” as short for “ $y \in X$ ” we get

$$(NC22*) \quad (\forall F)(\exists X)(\forall y)(y \in X \equiv F(y))$$

We can take this as the claim that to every concept (propositional function of the language, with the usual restriction on “ X ” in “ $F()$ ”) there exists an extension as a second-order *object*. (NC22) claims a correspondence between a type of function on first-order objects (i.e. concepts or propositional functions) as denizens of the second level and objects on the second level. This does not contradict (CT) making no cross-level correspondence claims. We can therefore combine versions of (NC) with (CT) as long as the versions of (NC) move extensions out of the range of the quantifier over the objects comprehended. Such a version of (NC), like (NC22*), will be stratified, but can still be impredicative. Frege, however, could not accept (NC22*): in (NC22*) we have second-order quantification, *and* the second-order quantifiers range *both* over functional entities (namely the concepts) and objects (namely the extensions). (NC22*) so violates one of Frege's most important distinctions: that between function and object. Frege's ontological dualism only knows objects

(including truth values) and functions (including concepts). Functions combine with their arguments to yield sentences. Two objects do not combine to a sentence. A sort of quantification covering both functions and objects generates thus syntactic non-well-formed expressions. This is unacceptable. If we want to keep the idea of extensions as higher order entities we have to introduce a second form of second-order quantification, say “ \forall ” and postulate:

$$(NC22') \quad (\forall F)(\forall X)(\exists y)(y \in X \equiv F(y))$$

We maintain thus the distinction between concepts and objects, but now one wants to know what our ontological model has become. We have a level of first-order entities (ordinary objects), a level of second-order entities taking first-order entities as arguments (i.e. concepts) and a level of second-order entities to which the first-order entities stand in the elementhood relation. These further second-order entities (being extensions) stand in the being-the-extension-of relation to concepts. Extension names are like ordinary names (singular terms) saturated (i.e. refer to an object), but refer to an object of another kind. They can be quantified over, but they cannot be comprehended with the objects into a single domain of quantification, on pains of introducing the contradiction (again). As they are objects one may suppose that concepts can apply to them. Frege postulated (DET): Concepts have to be determined everywhere. We can, of course, keep (DET) and all concepts can satisfy (DET) by being applicable to first-order objects. It seems we keep (DET) then only in letter and not in spirit: we have objects (i.e. extensions) now for which concepts are not defined. Syntactically, however, concepts better be defined not for them as otherwise they would have to take object variables of first and second order – syntactic garbage again. There can be mixed concepts, of course, relating first and second-order objects (like the elementhood relation, expressed by “ \in ”).

(CT) may apply to the domain of first-order variables, the powerset of that domain being beyond the reach of first-order quantifiers. In full SOL the members of this powerset are the values of the second order variables. The powerset itself is not an object of the theory. Given the preceding ontology with a second kind of second-order entities the members of the powerset of the domain of first-order variables are the values of the second-order object expressions. What about concepts then? What values to assign to them? Concepts may be taken not as sets/extensions but as functions, taking functions to be a *basic* sort of entity (i.e. not taking them as sets). One may also assign *both* concept- and extension-expressions members of the powerset of the domain of first-order quantification; they are thought to correspond anyway. The first solution, although not usual today, meets Frege’s insistence on distinguishing objects of any kind from functions. The semantics of the reformed system would contain then two domains of objects and a domain of functions/concepts.

On closer inspection, our problems reoccur. There should be concepts pertaining to extensions (i.e. our second-order objects). What about the extensions of *these* concepts? If they are again second-order objects (i.e. fall within the domain of quantification of extensions) we are back to our old troubles. If they are not second-order objects but objects of a further kind we are forced to introduce a hierarchy of objects (i.e. first a third domain of third-order objects as extensions of concepts of second-order objects, and so forth ...). [This reminds us of the fact that first-order ZFC can be carried out completely on first-order variables. So distinctions of size corresponding to (CT) occur within the domain of first-order variables.]

As GGA does not use third-order quantification one may just forsake higher order concepts. This, however, is both unnatural (as there are such higher order concepts even if not treated in one's system) and misleading anyway, as " \in " is already in use. What is the extension of " $a \in ()$ " for some constant " a "? It has to be a collection of extensions (i.e. a third type of object). Once we have this extension the story can be developed further and further.

Blame, so it seems, thus has to be laid on Frege's idea of an unstratified universe. Combining SOL with ZFC (i.e. ZFC2) allows distinguishing quantification of objects from quantification of functions/concepts. ZFC and ZFC2 come with the cumulative hierarchy as model for the domain of first-order quantification. The cumulative hierarchy (the universe V) itself is not an object of ZFC and the powerset of all sets of first order entities is not an entity of ZFC2. So a new ontological conundrum (or some variant of it) raises its head: What is the status of V ?

Frege's ontology maintained a universal set and avoided *this*, at least.

ZFC also abandons (NC) altogether in favour of the Separation Axiom (schema), and ZFC2 restricts comprehension to first-order objects. There are strong Fregean arguments in favour of (NC), however.

§16 A Fregean transcendental argument for (NC)

Frege committed himself to (NC12) as he is committed to extensions as logical objects. Extensions of uncontroversial basic concepts serve as first encounter with logical objects of the kind numbers turn out to be. Their claim to be 'logical objects' (i.e. being forced upon us by mere logic) rests on their close association with concepts: as we have a logical grasp of concepts we have a grasp of their extensions. From our logical knowledge alone we should expect that there is one extension for every concept – the very claim expressed in (NC). As logical objects are objects – and numbers are the very paradigm of logical objects – these extensions are *objects*, thus within the range of the first-order quantifiers, as (NC12) has it.

One can extract from Frege's philosophy a transcendental argument for (NC12), or at least for some version of (NC). The argument is 'transcendental'

as it ties (NC) to our ability to understand sentences. If (NC) belongs to the preconditions and background of that ability we cannot question it without raising doubts about understanding sentences, which quickly can prove self-destructive.

That arguments runs somewhat like this:

1. We distinguish sentences from lists of words.
2. There has to be something giving unity to the sentence.
3. This is the general term (i.e. the word expressing a concept) as it applies to singular terms by having at least one slot for arguments.¹³
4. The work done is not due to the ink marks or sounds but due to the reference of the general term: the concept.
5. The concept thus applies to the reference of the singular term to yield the reference of the sentence (as preserved under translation).¹⁴
6. In this the concept maps some objects to TRUE and some to FALSE.¹⁵
7. In the way of this mapping (extensional) concepts can be distinguished from one another.
8. This mapping is their *value range* (their graph).
9. Thus every concept has a value range (even if several general terms expressing the same concept can share a value range).
10. So, as we have to attest concepts as necessary conditions for sentential content (and structure), we have to assume value ranges.
11. If we define the collection of those objects mapped by a concept to TRUE as the *extension* of the concept, every concept has an extension.

Where should this argument be blocked?

If we do not want to endorse that every concept has an extension we need to claim either

- (i) Some concepts do their work of mapping without a map.
- or
- (ii) Some sentence like linguistic entities which appear like sentences are not sentences at all, since they lack a concept in their meaning.

Claim (i) is difficult to comprehend. It might be understood as claiming inaccurate maps not being maps at all, and thus ultimately may lead to a 3-valued map, which again would modify but not suspend the fundamental picture of

13 For the moment we only consider non-relational expressions.

14 We consider in this argument only reference, but a similar principle of compositionality works at the level of meaning (in the narrow sense of intensional meaning or Fregean 'sense'). The argument only gains strength in being non-committal about intensional entities, which play no role in GGA.

15 Adding further truth values (like INDETRMINATE) will not change the fundamental picture. Another truth value opens no real gaps in evaluation in any case [see next chapter].

concepts coming with a map. Even the concept mapping every object to FALSE is a proper concept.

Claim (ii) poses the difficult problem of distinguishing proper from apparent sentences. And even if this could be done (without ingenuity), we still have the situation that concepts are not to blame as they are absent from the meaning of these misbehaving linguistic entities. For concepts (NC12) still holds. Every concept (even ' $x \neq x$ ') has an extension then (even if it is \emptyset). One may question whether every sentence associates with a corresponding concept, but once a sentence is well-formed it contains a well-formed general term, which should at least correspond to the 'empty' concept (i.e. a map having \emptyset as extension). (NC22) can be understood as the very claim that to every propositional function there corresponds a concept, thus as an assumption which might be dropped. It is very difficult to see, how to force a wedge in here, especially in a standard (two-valued) logic context. If not every propositional function is a concept, then Frege's theory of the unity of a thought/sentence has to be overhauled substantially, again endangering the central claims concerning the semantic role of concepts.

If we have no better ideas (like accepting (NC12) and moving to a non-standard logic) the only way out is to deny straightforwardly that extensions are objects (first order entities in the basic case). We can then accept the transcendental argument, but deny the crucial step of including extensions into the domain of first-order objects and first-order quantification.

Frege took the idea of logical objects literally and considered them to be not just logical items of thought (after all a concept is something we think about and cannot be an object – the very idea of this bordering on the inexpressible in Frege's ontological framework, as witnessed by *Kerry's Paradox* of 'the concept horse'). Frege moves from 'logical object' to 'object' in the sense of being a member of the first-order domain. There are no other *objects* for Frege – again: since concepts are of a different ontological category. Frege also maintains no distinctions of levels in the realm of individuals. (NC) by itself is not inconsistent (not just with (CT) also by itself) but only in combination with a single sorted universe of objects. As we have seen [in §15], however, introducing a stratified universe means introducing the iterative hierarchy, which at least in its meta-theory contains inaugurating a further ontological categorical distinction: between sets and proper classes (or whatever might be the category V belongs to). Given his ontological categorical duality and the transcendental argument Frege had no option, but to fully endorse (NC12).

§17 The issue of abstract entities

The problems around (BLV) have been diagnosed sometimes as going back to Frege's plan to justify the assumption of abstract entities (cf. for instance Dummett 1991a: 209-40). Dummett traces (BLV) to Frege's strategy of employing his *Context Principle* to justify reference to extensions (and thus ab-

stract objects): as we understand expressions in the context of complete statements, so we are justified in assuming those entities (referred to by expressions in these statements) occurring in the truth conditions of these statements. The contradiction thus ‘refuted the context principle, as Frege had used it’ (Dummett 1991a: 225). Dummett endorses the *Context Principle* if epistemologically proper employed (ibid: 235-40). The explanatory failure goes back to the circularity explained above [in §14] of spelling out the truth conditions of sentences about extensions with sentences involving universal quantification about first-order entities thus referring us back to the extensions themselves (as supposed first-order entities). Well-founded epistemology has to proceed by first identifying the domain of first-order quantification and continuing with (DET) to extensions of concepts. The circularity prevents us from computing the identity of extensions using (BLV). This computational and epistemological failure *by itself*, however, does neither imply the inconsistency of (BLV) nor that the first-order domain cannot contain abstract entities. The failure shows the failure of Frege’s *epistemological* project to explain our reference to extensions as abstract objects. Frege’s epistemological project in the philosophy of mathematics rested less in establishing the existence of numbers, than in explaining *how* they are given to us. The *Context Principle* (especially in combination with definitions which equate the content of two sentences) linked numbers to more accessible content (like in HP numbers are linked to equinumerosity). The failure of this project due to the circularity involved [as seen in §14] does not show by itself the failure of (BLV) or (NC) as *ontological* truths.

Succeeding or failing to introduce numbers as extensions, in any case, does not exhaust the issue of abstract entities. Extensions go back to concepts as the prime logical entities [as we saw in §16]. Concepts are abstract entities and already the system of *Begriffsschrift* quantifies over them. (NC) is impredicative (the concepts comprehended may contain bound second-order variables) and does not develop the domain of second-order quantification piecemeal, but that only concerns constructivists – like Dummett. The system of *Begriffsschrift*, SOL without (NC12), is consistent, as Frege himself established there by arguing for its correctness. Thus, with the system of *Begriffsschrift* for a realist with respect to mathematical entities – like Frege – not only SOL is justified, but also taking concepts as abstract entities.

A ‘no class’-theory denying the existence of extensions (set/class-like entities) would be overkill on the other hand. The late Frege himself resorted to the idea that numbers are not extensions at all, but that numerals occur only within wider sentential contexts which create the impression that these were referring singular terms (cf. Parsons 1976).¹⁶

But even if it was feasible to substitute predication for elementhood statements and quantification over concepts for quantification over collections, which

16 A recent position resembling this attitude one finds in: (Hugly/Sayward 2006).

may not be easy to achieve, still there is the need for domains of quantification. There simply are collections of objects. If our symbolism cannot deal with them, so the worse for our symbolism. Most fundamentally our universe of discourse is a collection. Quantification essentially considers items collected to state something about *their* entirety or about some *out of* them.¹⁷ Once we acknowledge the domain of quantification it is nothing but natural – comprehension again – to acknowledge subcollections. There are kinds of entities. And kinds and collections are not nothings, so we should be able to talk about them, if not about *all of them*, then at least those we have concepts for. We arrive full circle at (NC12). With this we also justify second-order quantification, even if not full Second Order Logic.

[Using full SOL in the usual sense of being contrasted to SOL with a Henkin-style semantics, which reduces to many-sorted FOL (cf. Shapiro 1991: 70-76, 88-95), we would employ a formal system which validates inferences (like the ω -rule) which we as finite beings cannot draw. Thus we end up either as essentially logically incomplete (as our derivational system only captures a subsystem of the valid inferences of the logic in questions) or we have to possess a non-computational faculty of ω -rule like reasoning.]

§18 Believing (BLV) as axiom?

Frege himself famously reflects on the status of his axioms in the preface of GGA, and admits that the only axiom one may doubt to be a logical truth is (BLV), which he himself, however, believes to be a logical truth. The neo-Fregean debate around FA examined whether we should take some version of (HP) as analytic (cf. Heck 2011: 156-79). Is (BLV) analytic? This seems dubious as it engenders the contradiction (given standard logic as background logic). The transcendental argument above [§16] elucidates its intuitive appeal. What can it mean to believe it as an axiom? Frege himself stresses in the outlines of his general approach that all our confidence in our results has to rest in the correctness of the rules applied to logical truths as axioms.

Let us call logical, semantic and mathematical axioms which are not conceptual truths ‘structural truths’ (for lack of a better unifying term). ‘Conceptual truths’ are truths going back to the definitions of the concepts involved or going back to the meaning of undefined basic concepts. Structural truths, as to be explained, involve more.

Our cognition (‘mind/brain’) comes equipped with a conceptual framework; for well-known reasons the mind cannot be a blank slate (cf. Chomsky 1986). If within that framework some concepts have definitions we know *a priori* some conceptual truths, and we may re-capture them in our formal systems.

17 Plural quantification, if feasible, may reduce second-order quantification to (hidden) first-order quantification, but its semantics still presupposes subsets of the domain as collecting the entities of a kind, cf. (Linnebo 2003).

There is no problem with being certain about these conceptual truths as they repeat in object language terms some stipulations about the use of a term. But definitions, of the stipulative type (of the type considered appropriate by Frege in GGA) if non-circular depend on undefined basic concepts.

Circular definitions may be acceptable in some form of semantic holism, the circularity not being vicious if the circle is large enough and we do not have to follow it every time we use one of the defined concepts (employing some burden of proof routine to stop following definitional links). Given such a form of semantic holism all axioms may turn out to conceptual truths. As semantic holism of this type does not enjoy massive support – to say the least – we stick to Frege's picture of definition, and suppose that there are undefined concepts appearing in structural truths.

That some concept is undefined means we can give no non-circular definition of it. It does not mean that we cannot elucidate in some fashion how we employ that concept, and in which sentential contexts it is used. With respect to ourselves we may not need such elucidations at all – we just use the concept. In case of built-in concepts we come equipped with them as working cognitive tools. Only when reflecting on our concepts we might need to elucidate what to do with them. An example are basic propositional connectives. In stating their truth conditions we merely elucidate their meanings, because in stating the truth conditions we already presuppose these meanings. Stating a rule or a truth condition for a conditional, for example, presupposes being able to understand conditional reasoning. Given these basic meanings the axioms of propositional logic are conceptual truths. The same might be said about quantificational rules. So there are conceptual truths at the foundations of logic, but already definite descriptions provide a non clear cut case, as witnessed by quite different approaches to them (in standard or, say, in Free Logics). Frege's definition of definite descriptions depends on the prior introduction of value ranges.

Nothing forbids or excludes that our cognitive framework contains principles involving undefined concepts in non-definitional relations (i.e. containing axioms, traditionally labelled 'synthetic *a priori* truths'). If ϕ is such a statement we will be certain of it, as it is part of our cognitive equipment. Synthetic *a priori* truths are not true by meaning (as not being analytic). Their preferential status in comparison to empirical synthetic truths resides in their foundational character: they provide the frame for empirical theories. In semantic terms we have to say they are true in all models (all possible worlds) obeying the rules of our cognitive framework.

When considering a candidate *a priori* statement, how can we distinguish between conceptual truths and synthetic axioms or structural truths then? If we can look-up definitions in a lexicon they can be separated from structural truths. In case of a re-construction of our conceptual framework we may declare something as a definition to capture our mental lexicon. Using defini-

tions we have to ask whether they come with a specific epistemic quality to them. That quality might be a combination of (a) our certainty in them and (b) our knowledge of being able (in principle) to expand our definitional system, set the conceptual truth thus apart from the axioms. Conceptual truths therefore carry a mode of certainty that relies on our implicit access to our mental lexicon and the corresponding differentiation between defined and undefined concepts.

As there can be different modes of certainty we may well be able to distinguish structural truths from conceptual truths. Structural truths then are built-in principles we consider to be certain and not to be conceptually true. We are sure of them because they come to us as built into our cognitive framework. Once we acknowledge this thesis of built-in principles nothing depends anymore on claiming mathematics to be analytic. The logicist's claims concerning the analytic character of mathematics were important only given the prior rejection of the synthetic *a priori* truths. That rejection stemmed from a narrow conception of the synthetic *a priori* as involving intuition.

Thus the 'semantic tradition' (cf. Coffa 1993) went off the wrong track. Frege does not explicitly reject the synthetic *a priori* in GLA, and at the end of his life toyed with the idea of a geometric (intuition involving) foundation of mathematics. One should also not reject something as 'a priori' because one confuses 'a priori' with 'non revisable', since this is misleading as well. Of course we may be forced to revise our theories about what the structural truths are. To classify something as 'a priori' means to consider it to be a framework assumption in our currently best re-construction of our cognitive framework.

So, whether (BLV) and (NC) are analytic or synthetic does not decide about their foundational character. The intuitive appeal that comes with them may point to the fact that they are part of our cognitive framework.

If axioms are synthetic they cannot be re-constructed by conceptual analysis, there has to be more to the methods of philosophy. We may recognize an axiom by coming to see its role as an axiom of an area in question, founding the theorems there (in conjunction with definitions).¹⁸

In that sense (BLV) and (NC12) can obviously play a foundational role for set theory and mathematics, if only because their inconsistency allows deriving everything; but – to repeat – that inconsistency also depends on the background logic. (CT) in contrast (i.e. (CT) in the general case beyond finite set theory) *lacks* this intuitive support. As a theorem it draws logically and in terms of intuitive support on the Powerset Axiom. Cantor famously remarks on his first proof of the uncountability of the reals that he sees it but cannot believe it. It took sometime for Cantorian set theory to be accepted. (CT) carries *surprising* synthetic content. This might point to its not being *a priori*, but

18 Which formulas we take as axioms and which as theorems depends – besides the intuitive appeal a formula may possess – on general issues of methodology in developing formal systems, which we will enter in the next chapter.

only weakly so, as even conceptual truths may be surprising. (CT) in the powerset version relates to the Powerset Axiom. The Powerset Axiom has immediate appeal, even may be a conceptual truth, and follows from (NC) anyway. The crucial form of (CT), however, requires an additional ingredient: the Axiom of Infinity. The Axiom of Infinity is surely synthetic as it claims the *existence* of at least one infinite set:

$$(INF) \quad (\exists x)(\emptyset \in x \wedge (\forall y)(y \in x \supset y \cup \{y\} \in x))$$

This synthetic character already worried Russell as a supposed violation of logicism. The real conflict, therefore, may be that between Naïve Comprehension and the Axiom of Infinity (given standard FOL or SOL). *Finite* set theory supports (CT), and even given standard logic some version of Naïve Comprehension is *consistent* over the finite sets (i.e. if the comprehended sets are finite). [*Russell's Paradox* proves then that the *Russell Set* is not a finite set, which it should not be given the Axiom of Foundation. Note that the collection of all finite sets is infinite, but is not an object of the theory. Proving that every number has a successor (i.e. that for every set with some cardinality there is a set with an additional member), as required in Peano Arithmetic, proves that there is an unlimited supply of numbers, it does not prove that this supply is a set (of the theory).]

But the relevant version of Naïve Comprehension has to be (NC22'). If the domain consists only of the finite sets, then (NC22) is simply wrong, as not all comprehending collections are finite sets. If we modify (NC22) to

$$(NC22'') \quad (\forall F)(\exists x)(\forall y)(\text{Finite}(y) \supset (y \in x \equiv F(y)))$$

we have introduced a distinction between two types of collections *within* the range of one sort of quantification, thus, in effect, working with a way of comprehension that mirrors the Axiom of Separation (in this case separating subsets within the set of finite sets). (NC22') poses the problem of introducing collections which cannot be collected themselves in the formal system. And as finite sets can be members of finite sets the mere presence of “ $a \in ()$ ” may – *prima facie* – not immediately yield the severe problems discussed above [cf. §15] as the argument *can* be typed to objects of the domain of the first order quantifiers, but in this way “ \in ” becomes either ambiguous (and the second version of “ $a \in ()$ ”, with the argument being a second-order entity, does create the problems of §15) or we need a theory of correspondence between some second-order entities and first-order collections, resembling the theory of correspondence between some classes (improper classes) and sets in a system like NBG.

Finitism, thus – given enough ingenuity in its development – *may* resolve the conflict. $ZFC^{-\infty}$ is ZFC with an Axiom of Finitude (just the negation of the Axiom of Infinity):

$$(FIN) \quad \neg(\exists x)(\emptyset \in x \wedge (\forall y)(y \in x \supset y \cup \{y\} \in x))$$

ZFC^{-∞} can do what Peano Arithmetic does (e.g. proving Gödel's and Tarski's metalogical theorems).¹⁹ Finitism raises a couple of interesting questions, but may contain enough of mathematics for all practical purposes (cf. Bremer 2007). Given finite set theory we are left still (given standard logic) with the question what the status of the domain of sets is (the domain now being $V\omega$, in the light of the original iterative hierarchy), as that domain cannot be an object of the theory on pains of *Russell's Paradox* and its lot.

All of this, of course, does not decide in itself that (CT) is false and (NC12) true. In general evolutionary and pragmatist considerations support the thesis that built in cognitive assumptions are at least partially adaptive or associated (as saltations) with such adaptations and so fit to structures in reality. They are *a priori* only from our ontogenetic subjective perspective, but acquired phylogenetically. Our mind could come with principles which are not strictly true (i.e. true in all cases). Evolutionary and pragmatist considerations also support the idea that our cognitive framework has only to be as good as is crucial in the majority of our (cognitive) endeavours. Seen in this light a rule of thumb version of (NC) could be part of our cognitive framework, as the difficulties appear only in very specific contexts (usually of direct or indirect self-reference). Similar remarks could be made about general principles of a truth predicate and the *Liar Paradoxes*. Therefore the transcendental argument [of §16] may trace and support the embeddedness of (NC) within our cognitive framework, in this case our basic meta-linguistic knowledge of concepts. Even this, however, does not decide the conflict in favour of (BLV) and (NC) in their conflict with (CT).

§19 Lessons for Logic?

The conflict between (BLV) and (CT) leads to some major issues in formal ontology. It raises the interesting historical question how much Frege understood about (CT) and how seriously he took Cantor's diagonal proof. It seems that Frege did not recognize the force of (CT) and the obvious conflict to (BLV). Although the conflict commonly is settled to the disadvantage of (BLV) and in favour of (CT) and ZFC, the arguments in favour of (NC12) and thus (BLV) are, nonetheless, strong as ever. One option sees us turning to finitism. (BLV), (NC) and (CT) are compatible then. This option, however, deviates massively from standard mathematics, and certainly Frege would heap scorn on it.

The ultimate option to resolve the conflict lays in changing the logic underlying set theory or a theory of extensions. One option consists in endorsing (BLV) and (NC12) and changing the underlying logic of GGA to a paraconsis-

19 On further details cf. (Fitting 2007), on finite set theory in general see (Mahler 1968). Quine showed in his *Set Theory and Its Logic* that one can do arithmetic with the set of natural numbers being a virtual set only.

tent logic. One then uses (NC12) freely and assumes a flat universe, the universal set being an object of the theory. Seen from the arguments in favour of (NC) and the difficult meta-theoretic questions concerning the set-theoretic universe V this seems very advantageous. One may very well lose (CT) however, which loss – leaving us just with one level of infinity – will be the end of cardinal arithmetic. Paraconsistent set theories and inconsistent mathematics are too recent a development to see clearly where this leads and what part of standard mathematics can be regained (cf. Bremer 2010).

Preserving both standard logic and set theory (BLV) and (NC12) cannot be maintained – this, however, is just tautological. 'Revision of logic', on the other hand, sounds anathema given Frege's insistence on logic being the core of reason. But 'revision of logic' misleads anyway [as we will see in the next chapter].

Frege agrees to this:

- (i) For Frege the law of logic are not some conventions of some formal system, they are stated as basic laws in a (at least partially) formalized canonical representation of our (logical) knowledge.
- (ii) For Frege a crucial difference rests in logic being normative. Following proper logical rules helps to infer true (or in any other way designated) statements from the other true (or in any other way designated) statements.

Many logicians have suggested that some rules may be more appropriate in some contexts than other rules. Thus we come to see some formal system ('a logic') to be used on some occasion and not on an other. Logic thus seems up to choice. Frege does not agree, and given our theory about our cognitive framework we cannot agree either: choosing logic cannot be regarded as the whole truth for the simple, but fundamental, reason that in choosing some logic the mind cannot be a blank slate. Some core principles have to be operational in deciding on an applied logic.

Paradoxes can be considered as a heuristic to assess the coherence of a theory of logic, respectively its accompanying set of rules/axioms. A paradox or antinomy shows that a set of rules/axioms is not maximally coherent, has limited application. We find us in this situation with respect to (BLV) and (CT) given standard set theoretic assumptions (like infinitude), and standard FOL or SOL. Something has to give. This chapter argued that it is not obviously (BLV) and (NC12) which have to be given up.

Exploring Our Concept of Logic

This chapter tries to combine a version of realism about logic with a version of anti-realism about mathematics. Whether such an unusual combination is viable seems worth exploring, as usually anti-realism combines a – with respect to standard logic and mathematics – revisionist approach to logic and mathematics (typically in some form of constructivism).

The view developed here denies the very idea of 'revision of logic' (in some sense to be explained) and takes a structure like ZFC as the backbone of (pure) mathematics. Alluding to Carnap's famous *Principle of Tolerance* it claims that there is no room for tolerance in logic (in some sense to be explained), but a lot of room for tolerance in (pure) mathematics.

§1 Revising Logic?

When scientists get into trouble with their theories it is theories which are revised, not reality. If your biological account of an organ, say the kidney, provides no coherent explanation of the data you cannot revise the kidney, your account of its structure and function has to adapt. The same applies to the organ brain ('mind/brain' as is sometimes said).

Seen from this perspective the very phrase “revision of logic” has a misleading tone to it. Compare the case of languages: You can chose to talk German if you are able to when doing business in Germany; you can chose to speak Esperanto to impress your peers; but you cannot chose to have no natural language at all. Despite differences in approach and detail linguists agree that humans possess a language faculty, which is uniform species wide. The mind is not a blank slate. The language faculty has an initial state containing principles and parameters to be set. From this perspective (you may call it the 'cognitive science perspective' or the 'Chomskyan perspective') the same applies to logic. Humans possess – besides or as a part of – the language faculty a logic faculty or module that comes with a certain structure of principles. This structure is as it is, there is no room for 'logical pluralism' here.

Theories of logic share the fate of linguistic theories: they have to be revised if incoherent in face of the data. Theories of logic are revised, logic isn't.

§2 Theories of Logic

The questions to be raised now are: 'Are theories of logic in any way different from ordinary theories in cognitive science?' and 'What are the dimensions on which theories of logic should be evaluated?'

A crucial difference might be that logic is considered to be normative. Following proper logical rules helps to infer true (or in any other way designated) statements from the other true (or in any other way designated) statements. Some rules may be more appropriate in some contexts than other rules. Thus we come to see some formal system ('a logic') to be used on some occasion and not on another. Logic thus seems up to choice. Call this the 'logical positivist' or 'Carnapian' perspective on logic.

Choosing logic cannot be regarded as the whole truth for the simple, but fundamental, reason that in choosing a logic the mind cannot be a blank slate. Some core principles have to be operational in deciding on an applied logic. This core may be the logic faculty. Further on, normativity does not stand in conflict with explanatory theories. Compare linguistics again: Norms do not cease to be norms just because you describe their structure and give a (coherent) account of their function and what following them achieves.

Thus there is room for Carnap's *Principle of Tolerance* in choosing applied/regional logics, but behind and besides this we can study the core logic of the logic faculty.

§3 Criteria for Good Theories of Logic

After these preliminaries we have to turn to the dimensions on which theories of logic are evaluated, and what reasons may be given to prefer some set of rules to another set (which in terms of 'revision' can be read as: what reasons can be given to revise standard FOL).

In analogy to the general philosophy of science we have to look at the issues:

- What are the data a theory of logic has to account for? (To be considered are the issues of 'intuition', 'access', 'psychological reality' and reflective equilibrium.)
- What are the criteria of better coherence in case of a theory of logic?

Paradoxes can be considered as a *heuristic* to assess the coherence of a theory of logic, respectively its accompanying set of rules/axioms. A paradox/antinomy shows that a set of rules/axioms is not maximally coherent, has limited application.

In addition to meeting some standard criteria of coherence a theory of logic has to meet further criteria as being part of a comprehensive theory of cognition, like

- (i) feasibility (of the set of rules in complexity measures)

- (ii) being embeddable into a wide (partially evolutionary/naturalistic) theory of cognition (which raises, for instance, the issue of evolution going for working solutions in standard environments, not for principled solutions).

Lastly a theory of logic has to

- (iii) relate logic to epistemology and logic's function with respect to achieving epistemic virtues.

§4 Logical Data

What are the data for a theory of logic? On the one hand we can observe how people reason. Collecting examples and generalizing – maybe by disregarding supposedly obvious errors – one may thus come to a corpus of somewhat idealized ordinary argument patterns. (Generalizing and idealization are not completely unproblematic here, but no more than in other areas of science.) On the other hand a theory of meaning (for logical vocabulary of just for words in general) will come with a set of inferences based on meaning, thus being logical. Bringing these two sources together Nelson Goodman in *Fact, Fiction and Forecast*, John Rawls in *A Theory of Justice* and others have developed the idea of (wide) *reflective equilibrium* (cf. Terman 1993). The equilibrium has to take our intuitions of validity into account. Given a re-construction of the inferential rules and meanings (of logical vocabulary) involved, some of these pre-reflective intuitions can be superseded. Paradoxes (like the 'paradoxes of material implication') and antinomies (provable contradictions) play the role of abnormalities and recalcitrant data. A theory that can explain them away or accept them scores higher on the observational requirement of meeting the data than theories which do not.

Preferably capturing the rules of logic in some area of reasoning aspires to the following two ideals:

- (i) *Intuitive Correctness*: The inferences underwritten by the logical systems are intuitively valid.
- (ii) *Intuitive Completeness*: All the inferences considered to be valid intuitively can be derived using that logical system.

Within cognitive science the ideal of reflective equilibrium has been extended to the idea of *wide reflective equilibrium*: One has to consider not just our intuitive judgements of validity, but also constraints of cognitive (computational) complexity and learnability in a social or evolutionary context (cf. Stein 1996).

Another important constraint concerning the data basis of logical theory is

- (iii) *Accessibility*: All inference principles of the logical systems have to be cognitively penetrable.

In contrast to syntactic principles in linguistics, which are often or mostly processed sub-doxastically, rules of inference have to be accessible to rational

agents and speakers to some degree (have to be 'cognitively penetrable'). Rules of inference are employed and appealed to in communication and deliberation. Justifying assertions involves in principle the appeal to inferential procedures and standards of argumentation. These cannot be completely beyond the ken of the agents/speakers participating. Thus a logical theory postulating inaccessible principles can be ruled out. A logical theory containing gerrymandered or highly complex principles we cannot understand on first hearing is at least put in doubt.

§5 Simplicity of a Theory of Logic

General philosophy of science adds to the *observational requirement* of a theory fitting the data, preferably all the data (captured in the requirement of 'data completeness') criteria for a coherent structure of a theory. We consider here: *simplicity, explanatory power, consistency.*

§5.1 Ontological Simplicity

Simplicity comes as ontological simplicity and as methodological or structural simplicity, which is equivalent to explanatory power.

Ontological simplicity may concern either the number of types of entities allowed for in a theory or the number of entities (of some/any kind) allowed for in a theory. In the case of logical theories a contentious posit are possible worlds. Possible worlds have become common parlance in semantic model theory. One lesson to be learned here may be: As competing logical theories all employ possible worlds they are in the same boat with respect to that measure of coherence; criteria of coherence (and theory choice) can be indecisive in face of our best theories if they share the features related to these criteria of coherence; but if one theory stands out from the crowd of its competitors in that feature the scales can be moved in its favour.

Back to possible worlds: Suppose – unfortunately contrary to common practice – that the theories which employ possible worlds are clear about what they mean possible worlds to be. If “possible world” is only a title for some set theoretic structure we have only a case of wrong advertisement. The interesting case comes with the assumption of possible worlds as entities *sui generis*. In that case there seems to be an argument involving ontological simplicity available: Most theories that employ possible worlds already employ abstract entities (like sets). Models are set theoretic structures. If models are around anyway then models can stand in for possible worlds. Ontological simplicity decides in favour of models and thus against possible worlds. The argument can only be toppled by an appeal to explanatory power (i.e. that possible worlds are needed to explain semantic or logical features unexplainable otherwise). Typically (with the exception of David Lewis in *The Plurality of Worlds*) such arguments are missing.

Simplicity in the number of entities seems to be unimportant as most logical theories allow for an infinity of entities anyway. This need not be so, however, if some version of finitism can be sustained. The supply of expressions of a logical system need not be endless, but may be indefinitely large, so that in all practical employments of the system we never run out of expressions. If there are not infinitely many numbers (or what not else) then the logical meta-theory can employ finite set theory instead of standard set theory (like ZFC). Apart from dealing with finite collections only finite set theory has also the explanatory advantage of containing absolute complements and a universal set.

§5.2 Explanatory Simplicity

Simplicity in explanation (mostly considered as '*explanatory power*') is the key criterion of theory choice. A theory with simpler principles has more explanatory power as less or simpler principles cover the same ground as more or more complex principles do in other theories (given, of course, both theories fulfil the observation requirement). In case of logical theories theories involving less principles/rules or reduction sets (of logical vocabulary) may thus be preferable to those more complex. An interesting debate around that issue may be Michael Dummett's case for intuitionism. Dummett claims in *The Logical Basis of Metaphysics* that the intuitionistic rules for logical junctors and quantifiers are more appropriate than the standard rules as the intuitionistic introduction rules (in natural deduction) match the elimination rules; he states his case for some 'harmony' between these rules (some 'Harmony' with capital 'H' some 'harmony' without) as they are independent of each other, thus the rules for negation conservatively extending conditional logic, and so on. Dummett tries to establish 'harmony' as a new criterion to prefer a logical theory – as simplicity disfavours his account: Propositional logic can be reduced to a single logical junctor (say the Sheffer stroke). That one junctor allows to derive the complete set of propositional junctors, thus covering the maximal ground. It is much simpler to assume that the logic faculty comes equipped with the Sheffer stroke than to assume a set of junctors each independent of each other. Dummett appeals to theories of learnability, but an appeal to evolutionary theory may outweigh that: We can easily imagine that evolution equipped a cognitive system with the capacity to recognize that two things/states can not be the case together. With this standard propositional logic was in place. Any further developments might proceed from there, but have to use that core as point of departure.

§5.3 Consistency

Consistency was commonly – before the advent of paraconsistency – seen as a precondition for anything to count as a theory contender. A theory leading us into an antinomy is usually rejected. Even if paraconsistency (at least in the

form of dialetheism) allows for some contradictions being true not just any contradiction in one's logical theory are acceptable.

We have to distinguish here between a logical theory being inconsistent and a formal system allowing for inconsistency. Consistency works as a constraint in paraconsistency as strong arguments are needed to overrule that requirement (i.e. the presence of the Law of Non-Contradiction) within a formal system. One type of argument put forth by the dialetheists refers to simplicity: Some of our fundamental logical or semantic principles (like the Truth Schema or Naïve Comprehension) lead to antinomies, but these contradictions are acceptable as true contradictions, since these principles thus keep their maximal generality and simplicity, not to mention the failures of competing theories in that area (cf. Bremer 2005a). Another type of argument by proponents of paraconsistency refers to the observational requirement: Most people will not infer from some contradiction to any statement whatsoever; *ex contradictione quodlibet* is not underwritten by most people's logical intuitions. This means that logical rules that incorporate the *quodlibet* (like Disjunctive Syllogism or Modus Ponens) have to be understood as restricted in some fashion. In terms of formal systems this might mean that Modus Ponens has to be taken as a non-universal rule (like in a Default Logic or some Adaptive Logic).

§6 Feasibility

In addition to the general criteria of coherence a theory of logic inasmuch as it concerns cognition has to meet the further requirement of *feasibility*. Information storage and processing in humans is constrained by the general capacities of human brains and the affordable resources of deliberation in situated action. Results of computational complexity may not be easily transferred to human cognition (as complexity measures, for instance, work with worst case measures in the long run, where in applied cases an exponentially complex computation may be feasible on the usual input or a polynomial complex computation may involve too high a polynomial degree to be feasible on even small input). Nonetheless result of computational complexity theory might provide a rough assessment which rules of inference are more feasible than others. In case of alethic modal logics of necessity a further case can be made for S5 on basis of such feasibility reasoning. Propositional logic is NP-complete (by SAT being NP-complete). A modal extension of propositional logic which does not increase complexity of computing validity is *prima facie* preferable to an extension which increases complexity of computation. As S5 allows for reduction of modalities S5 is also NP-complete. In contrast weaker modal logics like K or S4, both of which involve many more basic modalities, are in a different complexity class: PSPACE. Thus S5 is vastly more feasible. Some modal logics that contain simple relational reasoning move to complexity classes EXP and EXPSPACE (cf. Blackburn/de Rijke/Venema 2001). A logical theory involving rules of exponential complexity has at least to add a supple-

mentary theory what (cognitive and computational) shortcuts may help to decrease this computational bottleneck (e.g. chunking statements or allowing for small error probabilities instead of certainty).

Having a proper theory of logic, and modelling the human logic faculty thus follows roughly the same methodology that other (empirical) theories of cognition do. As there is the human language faculty, there is a human faculty of logic. There isn't room for substantial 'logical pluralism' here (cf. Bremer 2013). As linguistic theories are revised to capture the initial state of the language faculty and its growth, also distinguishing competence and performance, so logical theories have to be revised in their attempt to capture in a formal system the core logic of the human logic faculty, and to account for a possible gap between the strength of that system and its pragmatic employment in situated deliberation and communication. Evolution might revise logic, logicians revise logical theories.

§7 The Inconceivability of the Creation of Logic

The preceding reflection on logic and the centrality of its core can also be turned into a theological puzzle. As an interlude let us therefore consider the inconceivability of the creation of logic.

Although in the last twenty years analytic philosophy has seen a rising interest in the philosophy of religion in general and in rational reconstructions of religion related arguments and Christian doctrines, the problem of logic has not received the attention, I believe, it deserves.²⁰

An old objection to cosmological arguments, named 'the Carriage Objection' by Arthur Schopenhauer²¹, charges them as being arbitrary: the arguments are employed to carry you to the existence of God, but no further (as the carriage carries you to some destination to be dismissed then, therefore the name of the objection). A simple cosmological argument claims the existence of the universe to require explanation, and offers God as the cause of the universe. The Carriage Objection now asks why the principle of sufficient explanation that carried the argument forth to God will not carry us on to a sufficient explanation of God, and then on – ad infinitum. The regress is considered to be vi-

²⁰ Quite a few anthologies and handbooks have appeared in the last few years. For instance: Stump/Murray 1999, Quinn/Taliaferro 1997, Petersen/VanArragon 2004. None of them (including Hoffmann/Rosenkrantz 2002) squarely confronts the problem discussed here.

²¹ Schopenhauer writes, discussing the cosmological argument, in *Über die vierfache Wurzel des Satzes vom zureichenden Grunde* (§20) that the principle of causality cannot be used 'like a carriage which can be send home after one has arrived where one wanted to go'. There is another passage to the same point in *Die Welt als Wille und Vorstellung* (p. 55) where Schopenhauer writes that the quest for an ultimate cause is dismissed after establishing God as the prime mover 'just like the bees kill the drones after they have served their purpose'. So one might call Schopenhauer's objection "the drone objection" as well.

cious. If one was to accept some brute fact (like the existence of God) then why not stop with the brute fact of the existence of the universe?

The objection has several weaknesses, as has been pointed out several times. For example, cosmological arguments from fine-tuning argue that the values of the natural constants are still in need of explanation, even if one takes the existence of the universe as a brute fact. As for the application of the principle of sufficient explanation some philosophers have claimed that it does not carry us any further, since God as metaphysically necessary is – in contrast to the universe as metaphysically contingent – not the type of entity which stands in need of explanation.

Whether these are good replies to the Carriage Objection will not concern us here in detail, what they presuppose, however, is a commitment to assumptions about God's Nature – as do several other arguments in the philosophy of religion (like God being wholly good, being outside space, knowing the past completely etc.).

With respect to God's Nature a problem arises that resembles the Carriage Objection. God's Nature seems to be something – a structure? – that is given even to Him. In the Middle Ages philosophers argued that God's inability to create the impossible (like a stone that not even He could lift or a proof of squaring the circle) is no objection to His omnipotence, since one must not demand breaking the laws of logic.

Where then do the laws of logic come from?

A dilemma raises its head:

The one horn sees the laws of logic as necessary in the strictest sense (i.e. at least metaphysically necessary or logically necessary in a sense even beyond that) and given with God's Nature. Arguments against God's omnipotence (because of the inability to create an unliftable stone etc.) do not go through then. Now, however, God seems to be limited by His nature (i.e. by finding Himself possessing this nature and not another). Further on, once we allow for God having just this nature to be a brute fact, we are again allowing for brute facts, and adherents of the Carriage Objection may stop somewhat earlier then.

The other horn sees God's Nature as being under His control. Some philosopher argue with respect to time that God committed Himself to be changeable by the creation of beings with free will (and thus unforeseeable actions), and thus changed one of His attributes. One may ask whether immutability was essential to Him in the first place then. In any case, however, a major problem remains: Even if He committed Himself to, say, *tertium non datur* by creating logic, there has to be some modus operandi by which He operates, even if His operations concern changing His own nature. One can hardly give up the idea that even God in achieving something uses some means (even if it is merely a thought of Him) to an end. And x being a means to an end y presupposes some minimal sense of x being operational in bringing y about. So there is a mode of operation of x. And then these mechanisms employed are beyond His con-

trol. The whole argument starts all over again, a vicious regress seems to loom here. If anyone goes along this path God's Nature seems to dissolve: One approaches some being (whatever that now may mean) with undifferentiated structure or nature, one departs from the God of Theism, not to speak of the Christian God.

§8 Ontological Anti-Realism

My (psychological) realism about logic is realism about *representations*: rules of logic are representations, inferences are ways to process representations, logical structure structures representations. Logic deals with inferring representations (mainly sentences) from other representations (mainly other sentences or their conjunction). Logical realism of this kind fits into a Representational Theory of Mind. Logic need not commit itself to the existence of propositions besides representations (as witnessed by treatments of logic by logicians like Quine who do not believe in propositions).

Mathematics, on the other hand, seems to come with unavoidable massive ontological commitments, which carry over to logic, once its meta-theory is cast in model theory. Again one may argue that our conceptual scheme contains some basic mathematical concepts, prominently some concept of collections like extensions of concepts, sets or heaps. Completely different accounts might be given of these: like a Fregean theory of value ranges for extensions, some mereology (extensional [with Lesniewski (1929)] or intensional [with Simons (1987)]) for heaps, and some set theory (ranging from finite set theory to ZFC variants, to variants with sets and proper classes like NBG or MK, and many other versions like KP, NF and what not). In the vain of the discussion above one may ask which are the principles our ordinary concept of collection relies upon. Controversial, but supposedly obvious, candidates are Naïve Comprehension, Frege's Basic Law V, Existence of General Sums ...

One may doubt, however, whether evidence for one of the complete systems can be put forth. And one may now ask oneself how we have to take the ontological commitments that come with these systems. Therefore we turn to consider ontological anti-realism. Ontological anti-realism in mathematics turns out to be compatible with (psychological) realism about logic and basic mathematical concepts.

Any form of anti-realism in mathematics – just as any form of realism in mathematics – has to account for

- (i) the meaning of mathematical language
- (ii) the *a priori* nature of mathematics
- (iii) the applicability of mathematics to reality in the empirical sciences.

Supposedly (1) poses the greater challenge for the anti-realist, than for the mathematical realist, whose challenge is (3).

§9 Mathematical Conventionalism

On an anti-realist view mathematics is *objective* by being true only by force of the conventions laid down. Mathematical truths are derivable, and nothing but derivable. An anti-realist in mathematics *identifies* truth in mathematics with derivability. Truth in pure mathematics is 'true in the story of pure mathematics', coinciding with being derivable from the axioms of pure mathematics by the rules of pure mathematics and logic alone. Thus mathematics is also *a priori* and analytic. This answers question (2) above. Difficult proofs may enlarge our knowledge and deepen our understanding of the impact of the conventions, so even analytic sentences can be subjectively surprising and be a gain in explicit knowledge.

Pure mathematical talk has *meaning* by the conventions of pure mathematics (i.e. rules of usage and recursive truth conditions). This is half of the answer to question (1) above.

This conception of mathematical truth stands in no conflict with Gödel's Incompleteness Theorems, as one may observe (following Wittgenstein [1964]) that the reasoning establishing the truth of the Gödel sentence takes place in *another* formal system than that in question, as well as claim (following Dummett [1963]) that the non-coincidence of truth and provability in some system only shows that our intuitive resources of reasoning – as employed in the meta-reasoning – are not completely formalized, and so the respective system may be extended indefinitely, or one may even (following Priest [1987]) derive the Gödel sentence for a paraconsistent system in that very system.

§10 Mathematics as Fiction

The ontology of pure mathematics (i.e. pure set theory) supports this objective quality by providing a picture of independently existing entities warranting and corresponding to the objective mathematical truths. This realm is a *fiction* accompanying the conventions of mathematics. To answer question (1) completely with respect to reference of expressions in pure mathematics one thus adds: we are presented a story/a picture of a realm of entities which serve as substitute referents for expressions in pure mathematics the way fictional characters serve as substitute referents for their names, which means properly speaking they *do not refer at all*, but are 'mere' representations. Pure mathematics tells a story, but not a story *about* something, neither about the 'forms' of Platonism nor the 'non-existing objects' of Noneism.

Pure mathematics, however, is distinguished from other fictions (other arbitrary conventions) by its applicability in the sciences and everyday life. One may account for this – and so answer question (3) above – as follows:

- (i) Pure mathematics (i.e. ultimately pure set theory) consists of a linguistic structure containing both expressions (supposedly) referring to single entities as well as those (supposedly) referring to relations and properties.
- (ii) Parts of reality (e.g. countable objects and their measurable properties) provide *a partial model* of this mathematical structure. A model in the not set theoretic sense: these parts of reality can be linked to mathematical expressions (e.g. in case of measurable extents of qualities) *and* the derivable consequences with respect to them (derived using the purely mathematical structure) *apply again* to parts of reality.
- (iii) This homomorphism (in the non-technical sense of the consequences of the picture being a picture of the consequences) invites us to assume that those parts of the purely mathematical language we have not fixed to some part of reality may nonetheless be understood as having a model, because we believe that we just have not observed their counterparts yet, or we treat their counterparts like theoretical entities in the sciences, or we just don't care about these counterparts as long as the homomorphism stays stable on the observed counterparts. One may take these parts of pure mathematics as useful supplementary fictions. They are 'supplementary' as there is nothing in the partial model relating to them.

The fictionalist with this relates *the whole of mathematics* to reality by anchoring pure mathematics in a partial model of it. In this view there is some truths in a realist picture which sees mathematics corresponding to structures of reality. In this view non-applied (purely pure) mathematics can be tolerated by its service to applied mathematics. It shouldn't be taken as exploring a self-sustaining mathematical reality. We can even speculate that our conceptual scheme rather contains a mereological concept of collection, which is taken up and extended (e.g. by postulating the existence of an empty set, or by adding the assumption of singletons [cf. Lewis 1991]) by set theory. In that case the conceptual support for our immediate judgement about set theory and its principles may *rest in (intensional) mereology*.

Examples: If you have a measuring rod and you can place it exactly alongside a piece of wood that piece of wood has length 1. If you believe that another piece of wood has length 3, then because arithmetic tells you that $3 = 1 + 1 + 1$, you should be able to place your rod alongside that other piece of wood, make a stroke besides the piece of wood where the rod ends, put the rod alongside the piece of wood again starting from the stroke, make another stroke put your rod alongside that other piece of wood again, and achieve a match in length. In the second example using arithmetic on the arithmetic description corresponding to the non-arithmetic description one *derives* an arithmetic description which again corresponds to the non-arithmetic descrip-

tion of the second case, thus exhibiting the homomorphism of the partial model.

§11 The Story of Mathematics

Pure mathematical talk fulfils, thus, another function than scientific talk. Scientific sentences are true or false, including those containing mathematical elements, as these elements are linked to procedures of establishing the truth value of a sentence (e.g. by the use of measuring paradigms). Scientific language aims at the facts in describing reality. Pure mathematics, in contrast, can be seen as either 'true to the story of mathematics' (by being derivable given the axioms) or as just *telling and establishing* the background story to applied mathematics (being objective by its intersubjectively shareable character of conventionality). The language game of pure mathematics doesn't concern the description or denotation of mathematical facts.

An overall fictionalist account of pure mathematics claims that even pure mathematical sentences talking about finite cardinals do not refer to mathematical facts, not just those purely pure mathematical sentences dealing with the remote regions of *Cantor's Paradise*. Scientific or everyday assertions containing numerals (e.g. "There are 3 apples on these 2 tables") possess truth conditions or are linked to procedures of justification that do not involve *numbers*, even finite cardinals. Procedures and rules covering counting, measuring or employing a ruler and a pair of compass serve as bridge principles relating empirical sentences and observations to pure mathematics.

Thus the fictionalism outlined differs from anti-realistic constructivism, which at least maintains those parts of pure mathematics which have been constructed. The fictionalist doubts the use of the constructivist's further assumption that by carrying out steps of reasoning inside mathematics we have *supplied further entities*.

Besides fictionalist anti-realism there are more well-known forms of non-standard treatments of mathematics, many of which have a constructivist streak, often including their treatment of logic (cf. Tennant 1987). One may, for instance, target the understanding of universal quantification. Contrary to Cantor's Domain Principle, which assumes the existence of a domain of values for the variables quantified over, one may understand universal quantification like a conditional (substitutional) claim: once a value (or a term) is provided the universally quantified sentence holds of it, how many whatsoever these values (or terms) may be. One need not even take the domain they are supposedly are collected from to be completely given: it might be expanding or be otherwise elusive, what counts is only the conditional claim on any values (or terms) provided. This resembles using schemata with schematic expressions instead of variables and quantifiers. Such an account trades in a non-standard use of quantification (in mathematics) for ridding itself of a fictionalist account. A fictionalist account of pure mathematics need not involve a revision-

ist understanding of quantification, not more than any account of fiction, as fiction in general also employs quantifiers (be it counting fairies or numbers). This fictionalist anti-realism does not 'revise' standard mathematics (or ZFC) in any sense, e.g. by changing its underlying logic. The anti-realism does not pertain to logic, but to ontology.

Identifying something as pure mathematics serves as a rigidly syntactical *indication of scope* which puts the assertive force of the involved declarative sentences into brackets. The inscription “a novel” on the title page or cover of a book informs us that we confront a work of fiction, we put what is said into the brackets of a story – almost the same applies to “a treatise in set theory”.

§12 Pluralism in Mathematics

In as much as reality only provides a partial model of mathematics reality cannot distinguish between those mathematical structures which are equivalent with respect to the descriptions and the projections covering the partial model. Therefore more than one mathematical structure can be applicable to reality, and thus be useful, and in this sense be justified.

This will be so for set theoretical differences (say in large cardinals) way beyond any direct relation to applied mathematics. Postulates of the existence of large cardinals may either be rejected as superfluous or may even be endorsed as equipping the complete mathematical structure with valuable structural properties like symmetry or non-arbitrariness, or implying theorems deemed valid (like V not being equal to the ‘constructive universe’ L). This may also be the case for the distinction between mathematical structures differing in the *cardinality* of the number classes involved (i.e. those being finite, enumerable or more than enumerable). Reality may not be – or supposedly is not, according to quantum mechanics – continuous, not even dense. Not just the rational and real numbers may be too much – even large *finite* cardinals may have no application to reality.

So, even if there is no conventionalism with respect to logic in as much as our logic faculty is concerned, there is plenty of room for *conventionalism in mathematics*. Quantifiers (of some sort) and thus some powers of counting objects are part of our logic faculty, set theory (say in the form of ZFC) almost certainly isn't. Carnap's Principle of Tolerance applies here, as well as meta-theoretical criteria of theory choice in pure mathematics (like symmetry or ease of computation).

§13 Two Types of Paradox

Insofar as pure mathematics (set theory) serves only as the background for applied mathematics and carries no ontological commitment by itself, we needn't be as concerned about set theoretical paradoxes and foundational problems as mathematical realists are. A story may contain unsolved puzzles or even con-

fusions – they do not matter as long as they do not affect those parts of the story relevant to us. A novel, for example, may contain errors and confusions concerning the economy of a society depicted, which nonetheless may be irrelevant to its main plot (of character development or crime detection). In that vain foundational issues in set theory lose a lot of their interest to the mathematical pragmatist, quite contrary to the semantic and logical paradoxes, which highlight either an insufficient re-construction of our logic faculty or even inbuilt conceptual mismatch.

The focus on applied mathematics as crucial explains the lacking interest of the working mathematician in pure mathematics. If there were more than lip service paying real mathematical realists there should be much more concern about the problems of set theoretical foundations (like the status of the universe V or the existence of \emptyset). Working mathematicians by their pragmatism embody an anti-realistic attitude to mathematics.

Fictionalist anti-realism can take some ontological riddles and some paradoxes at face value and integrate them into the story of set theory. It even leaves behind the scruples of fashionable ‘axiomatic ontology’ which stipulates the existence of entities and their kinds. One may illustrate this by the case of the set theoretic universe V .²²

The idea that there is no universal set seems to go against our logical intuitions as we have developed them working with quantificational logics: There is always a *domain of all objects* to be quantified over. What then can be the semantics of ZFC? How are its quantifiers to be understood? Although there is no universal *set*, there is *universal quantification* in ZFC. The axioms witness this. The Axiom of Separation, for instance, says of *all sets* that for any condition the corresponding subset exists. In terms of the iterative hierarchy the axiom talks about sets *of any rank*. The set theoretic universe V is the range of the quantifiers in ZFC. Cantor claimed that every potential infinite presupposes an actual infinite ‘and cannot be thought without it’ (cf. Cantor 1887). This is the Domain Principle: Speaking of and quantifying the x presupposes the domain of the x . As sets are abstract, eternal entities, given the story of set theory, they do not depend in their existence on any one counting up to them. Sets are simple *there*. *All of them* are there. The metaphors of construction employed in telling the story merely serve to express the structures the sets employ, and may serve, sometimes, as didactic devices how we come to understand some set on the basis of another collection of sets. Thus, that there is no highest rank in ZFC should not be misunderstood as V being under construction. All sets are there, thus V is *there*. V is not reached by any ladder (‘construction principle’) used within it. It is as strongly inaccessible by such steps as it can be. Otherwise we only have a temporary halting point, namely a

²² Cf. Bremer 2010, where, unfortunately, fictionalism of the kind discussed here is not explored as an option. [Remember: The issue of a universal collection is not bound to set theory. A similar story could be told for the collection of proper classes in NBG.]

rank V_α . V is no number, is no set, no union or power of sets. V can only be thought as *sui generis*. How do we know this? Otherwise it could be superseded in one of the usual ways of set access. We thus have a *transcendental argument* concerning V 's nature: it cannot be otherwise, since otherwise it wouldn't be. Calling V '*sui generis*' only gives a name to the riddle, of course. Here, however, the *story* of set theory can enfold even more. It tells us first that V is a very special entity, both within the picture of the iterative hierarchy as in our meta-theory modelling our theory of sets. V has no subsets as V is no set. V is not well-ordered. V is not the domain of a (replacement) function: sets are – and so on. V contains all ordinals and all cardinals, but there is neither a set of all ordinals nor a set of all cardinals. They cannot be established as subsets of V , since V is no set (and thus Separation does not apply to it). To this the fictionalist can add some claims which non-fictionalist have to read as (ontological) postulates of a controversial nature: V is *determined*, not indefinite, and *unique*. Formally this means that V cannot be an element of whatsoever other collection (on pains of re-introducing distinctions of the set/class-type), that there are no other entities of V 's type (not a collection of proper classes), that V is an entity which can be talked about by its name, *without including it into a domain of reference*. V is not a standard object of (set theoretic) model theory. The only thing V 'does' is containing all the sets. A universally quantified sentence of pure set theory is meaningful, in our set theoretic story, as there is an entity which provides all the variable values: V . A unified set theoretic language has to distinguish urelements, sets and V . Again: V cannot be unified with them in a domain. The name " V " refers to V rigidly. End of story.

§14 Psychological Realism and Ontological Fictionalism

Psychological realism with respect to logic and pragmatism with respect to mathematics are compatible, as the logical realist stops at the existence axioms of pure mathematics (especially the Axiom of Infinity). Even a dose of *logicism* may be compatible with anti-realism in mathematics: it may be so that our logic faculty (i.e. our conceptual scheme with respect to logical concepts) allows for the derivation of some advanced mathematical concepts and structures. Realism with respect to logic meant that we *have* this one very logic, it does not mean that all concepts employed in that faculty have objective reality in application. As often with human cognition they only have to be good enough in our dealings with reality. In exploring our concept of logic we thus also outline its inbuilt misconceptions if any. As scientists we then proceed to correct those misconceptions by proposing conceptual reform. Although we thereby aim at a proper concept, say of opposites and negation, we got a foothold on that by starting from an analysis of our concept of opposites and negation. Only after having exhausted these given conceptual resources amend-

ments can be justified. Even recording our misconceptions contributes to our anthropological knowledge, at least.

Thus the concepts of our logic faculty may invite and sustain some elaborate fictions of pure set theory that underwrite pure mathematics. Again one has to separate immediately applicable mathematical talk of entities, structures and consequences from scaffolding. One may accompany one's psychological realism with respect to logic, of course, with a psychological realism with respect to our numerical faculties (involving, say, concepts of collections, division or successor). The formal reconstruction of the concepts involved there, however, will presumably not attain the mathematical power we find in set theory, as witnessed by the problems of providing a purely mereological foundation of current mathematics.

Realism with respect to logic rests in the fact that we possess a logic faculty that is the way it *is*. Anti-realism with respect to mathematics rests in the belief that there are neither pure sets nor numbers of any kind. This fictionalist anti-realism answers the question how a nominalist can live with set theory, although she does not believe in sets. There are no facts of the matter to be discovered about them. There are matters of fact concerning our logic faculty. Our best theory of logic systematizes them.

§15 Comparative Remarks

The foregoing fictionalism differs from those versions of fictionalism which advance the thesis that mathematical claims to truth are to be taken at face value and thus *are just false* (cf. Kalderon 2005). Calling fiction "false" misleads inasmuch as fiction does not aspire to be true. There are often individual true sentences in a fiction (say that Baker Street lays in Marylebone), but the whole story is told within the scope of an indicator of fiction. Mathematics as a whole is neither true nor false, parts of it (those anchoring the partial interpretation of mathematics in face of reality) correspond to true sentences (taking about relations and extends of properties).

The foregoing psychological realism also differs from those theories advanced by adherents of 'Relevance Theory' or Grice's theory of implicature or 'filtration logics' (cf. Schurz 1991), who all take ('classical') standard logic as the logic operative in our cognition, which first delivers all kinds of inferences like *ex contradictione quodlibet*, which are then not taken or not uttered, because of pragmatic filters. This would be a waste of computational resources and needed cooperation between derivational and pragmatic modules – which makes an approach that starts with a more restrictive non-standard logic as psychological real seem more promising.

The perennial problem how we access mathematical entities is easily solved by accessing the fiction. There is nothing else to access.

The Inexpressibility of Real Gaps in Ordinary Language

There is something wrong with the concept of truth value gaps when applied to natural languages.

Many 3-valued approaches to the antinomies can be rejected as insufficient, because they either face strengthened versions of the antinomies or because they *employ* distinctions in semantics which they *cannot express* themselves. They are applicable only to partial languages and not to inclusive languages like a natural language, which has to have the resources to express any semantic distinction, as the arguments introducing these distinctions are expressed in natural language or a canonical regimentation of it. Such gap theories fall prey to Strengthened Liars and ‘Revenge’, which reintroduce the bifurcation needed for the antinomic reasoning (cf. the introduction and the papers in the volume *Revenge of the Liar* [Beall 2007]).

Pragmatic approaches to the antinomies often suffer from their proximity to such 3-valued approaches.

In this paper I explore the idea of gaps in evaluations again.

[Note on usage: an ‘evaluation’ assigns a truth value, an ‘interpretation’ assigns meaning, so that there may be an interpretation without an evaluation.]

The crucial idea to be tested rests in the assumption of a (meta-)semantic rule which can enforce the retraction of a supposed evaluation of an antinomic sentence or statement. Such a rule belongs to pragmatics inasmuch as it regulates the derivation or rejection of an evaluation of a sentence in a context of assertion. It regulates the communicative role of some sentences.

We start, however, with a preliminary discussion of an attempt to deny the existence of token-reflexive truth value bearers in ordinary language.

§1 Token-Reflexive Sentences and Statements in Ordinary Language

The word ‘this’ can be used as a deictic expression roughly synonymous with ‘that’ or as a strictly reflexive indexical. So

(1) This sentence has five words.

can mean – in one reading to be disambiguated –

(2) That sentence (over there) has five words.

or it can mean – in another reading to be disambiguated –

(3) This (very) sentence (I am using right now) has five words.

In expositions of the antinomies one often uses ‘this’ in the sense of (3), e.g.

(4) This statement is not true.

As the ‘this’ is strictly reflexive its reference does *not* vary from occasion to occasion (of using sentence (4)). It always refers to (4).

Statements result when a speaker uses a sentence on an occasion. Sentences often are indexical and thus are neither true nor false as they stand. The indexicals have to be anchored to a situation of usage. *What is said* on that occasion then can be true or false. The distinction has found use not only in pragmatics, but is part *inter alia* of the framework of situation semantics. Statements are eternal (cf. Strawson 1952). Statements can be presented by *eternal sentences* or by some abstract representation (like ‘infons’ in situation semantics or state of affairs in other ontological frameworks).

In these abstract representations any indexical part present in the sentence used (say “I sit on that chair”) is replaced by the item/referent talked about in the statement: “I” is replaced by the speaker, “sit on” by the relation of sitting on, “that chair” by that chair, including times and places, and so forth.

Therefore: An occasion of using a Liar sentence gives rise to a Liar *statement*, which is eternal, and refers to this very statement, and the property of truth, and so forth. For the statement (i.e. this objective content, which could also be specified as a tuple of items including that very statement) it is of no importance how the items are referred to.

[As statements can at least be expressed by eternal sentences I follow the practice of speaking of (eternal) sentences as truth value bearers. And I take eternal sentences as expressing the *content* conveyed by using some sentence in an assertion (in a situation).]

Thus when someone reasons about (4) that (4) has no semantic value, and thus is not true, he *is not making another statement* than (4) does (or somebody using (4) does). He can speak French or whistle, as long as some conventions of fixing reference are in place he is making the same statement. Of course his speech act takes place on another occasion than mine, two tokens of sentence (4) are used. But he makes the same statement that anybody makes who uses (4). The “this” in (4) does not indexically refer to some sentence used in a situation of usage, but refers strictly reflexive to (4). And as his reasoning leads back to the statement made by (4), the antinomic reasoning proceeds as it always does. One may be misled by the usage of ‘this’ or other natural language renderings of the antinomies, but shortcomings of phrasing the antinomies in these ways should be surmounted by formalization.

Some of the problems concerning whether some sentences really ‘say’ something or not might be solved by trying to be more precise on the formal side of the sentence/statement distinction. Unless one believes that natural languages are beyond formal treatment there have to be some formal means or representations which capture that distinction. Logicians exploring the logic of asser-

tion and rejection sometimes use the syntactic derivability sign (i.e. \vdash) or some new device (like: \vdash) to capture assertive force. Assertive force can be represented in natural languages themselves, which, of course, pretty nicely explains that we can write papers about it.

Now, as the sign of assertive force might define what distinguishes a statement from other ways of using sentences, and as this operator/sign is part of the formal machinery of language, one can distinguish well-formed formulas containing it at the proper place from others. Thus we can define the class of statements (or statement representing eternal sentences). As this is a syntactic identification, the process of doing so is primitive recursive. Therefore we have all reasons to believe that the antecedent conditions of the *Diagonal Lemma* are fulfilled, and thus *that there is some sentence λ equivalent to the sentence that asserting λ results in a false statement*. As no indexicals are involved that sentences λ not accidentally is usually taken as the eternal sentence representing the Liar (statement). Once we have that eternal sentence in the context of an assertion sign it seems to be pretty *ad hoc* to deny that it says what it says. Even if no one bothers to utter it right now someone may do so, and λ just has the objective content it eternally has.

There are, thus, token-reflexive sentences in ordinary language. Even if “this” in (1) usually refers to another sentence (has an unspecified argument position), we can substitute a name of the diagonal of that sentential function into that position thus arriving at a token-reflexive sentence. From a syntactic point of view ordinary language seems to contain token-reflexive sentences. Otherwise ordinary language wouldn't be as expressive as some artificial languages.

§2 A Semantic Rule for Gaps in Ordinary Language

How are we to interpret (4) then? Given standard semantics we arrive at the antinomy. – Can we avoid interpreting it at all? Even if we cannot make sensible use of (4) [see below], can we suspend (4) from having an evaluation? Given mildly realistic assumptions in semantics, even if we personally do not care about the sentence it has an evaluation, thus ordinary language will be inconsistent, whether we care or not.

One might resort to some meta-rule on evaluations that revises all evaluations in case they land us in contradictions.

Firstly, such a rule will be beyond standard logic towards default or adaptive logics (cf. Batens 2000 on Adaptive Logics, Besnard 1989 on Default Logics). Secondly, if we do not distinguish object-language from meta-language this rule and its negation may enter into reflexive constructions. In contrast to a sentence, however, if there was a corresponding reflexive rule generating contradictions we simply need not adopt it. A sentence saying of itself that it was not subject to the original rule might be without evaluation itself.

The meta-rule had to say something like

(MR)

Whenever a sentence *A* on a supposed interpretation and evaluation (i.e. a straight forward application of the semantic rules) leads to a contradictory evaluation, *A* has no evaluation. The supposed evaluation is rejected.

The rule enforces and says in effect that we will never accept contradictory evaluations. It relativizes the coverage of our semantic rules, which are otherwise understood as applying to all sentences. It is a rule about semantic rules. Given (MR) sentence (4) in fact has no evaluation. Relatives of (4) which token-reflexively say of themselves that they have an evaluation and that they are false are not simply false (as this would be an evaluation), but lack an evaluation themselves. (MR) regulates the absence of interpretations in general, even involving sentences referring (indirectly) to (MR), like

(4') If this has an evaluation, it is false.

(4'') This has an evaluation and is false.

Such sentences are without evaluations themselves. (4'') is antinomic if evaluated, so one just leaves it without evaluation.

“This has no evaluation” can be consistently interpreted as false. So it seems that (MR) allows for consistent self-regulation and does not face a Strengthened Liar, so far.

Do sentences like (4) have a use in ordinary language?

In indirect speech we can, of course, mention them as examples of problematic sentences. In a straight forward assertive utterance the felicity conditions of such an utterance cannot be met, given (MR), as we assert as true a sentence which cannot be evaluated. In a liberalized understanding of the felicity conditions we may even allow for such utterances inasmuch as they succeed in putting forth a sentence to be evaluated, but are followed then by an act of retracting the assertion. The speaker might be blamed for violating the rule of asserting only what one believes to be true, or be blamed for believing this sentence to be true (i.e. for incomplete reasoning with respect to the sentence or neglect of [MR]).

In any case, showing that we can achieve no purpose by using (4), or even showing that an attempted assertive utterance will misfire, does not by itself entail that (4) has no evaluation. We need the additional appeal to a meta-rule like (MR). In this respect the treatment of the antinomies in ordinary language explored here is *not* just pragmatic. It concerns our framework of semantic rules.

Further on, that (4) has no assertive use and receives no evaluation need not and should not entail that (4) has no interpretation, no meaning. We understand what (4) wants to say, because we employ our capacity to understand token-reflexive sentences. Many of them are harmlessly true, like “This is a token-reflexive sentence”.

We understand (4) inasmuch as we know which semantic rules are to be applied to it (compositionally). We derive an interpretation. Once we arrive at the supposed contradiction (MR) will be activated and no evaluation is delivered on (4). The real gaps are gaps with respect to the evaluation of sentences, they are not semantic gaps, say in the sense that some sentences have no meaning.

This whole approach looks like a semantics with truth value gaps (or even interpretation gaps), and then a Strengthened Liar

(5) This is not true.

should land us in a new version of the old antinomy, because *being* not true by falling into the *gap category* seems to make (5) true, and we have regained an antinomy. Having *no* evaluation cannot be just another evaluation (included in the opposite of being true), then, without contradiction. Gaps must be *real gaps*. They have to lead to real gaps in evaluating discourse and to real gaps in drawing inferences. The semantically crucial issue is distinguishing the statement that (4) is not true, as it has no evaluation, from an evaluation. So far we have no theory how such 'real gaps' should work. Suppose, for the moment, that gaps do not fall prey to a Strengthened Liar. Nonetheless we have to be able to *talk* about them.

§3 The Inexpressibility Argument

One way one should not try to circumvent a Strengthened Liar is by restricting “not” in (5) or related sentences to be some kind of 'inner' negation which carries us only from truth to falsity and from falsity to truth. This way we could not even express that sentences like (4), which have no evaluation, are therefore *not* contradictory (i.e. true and false at the same time). To express the theory of gaps and (MR) we cannot forsake a 'wide' negation (cf. Bremer 2008a).

Meta-linguistic sentences talking of the predicate “() is true” also do not change the picture, as the old antinomies can easily be transferred into meta-linguistic versions like

(5') The predicate “() is true” cannot be successfully applied to this very sentence.

which raise the same issues of contradictions or inexpressible semantic facts. [Left as an exercise to the reader.]

We face problems of inexpressibility with our treatment of (5).

Saying of (5) that it has no evaluation should be true. Suppose it is. Then it *is* also true that (5) is not true (as being 'true' would be an evaluation). And it would be a limit of semantic expressibility if we could not say so. Suppose we could say so. Does *this* then *imply* that (5) is true after all? – No! No, because to derive this claim we have to apply some semantic rule for “() is true” to (5), the application of which will be retracted by (MR). How can we say then

that (5) is not true? *Saying* that (5) is not true seems like an instance of the type of (5), at least we seem to make the same statement like (5), as discussed in §1. That saying of (5) that it is not true has meaning (an interpretation) seems to be not enough, since we seem to know that it *is* also true. Saying of (5) that it is not true should be evaluated as being true, but this brings us full circle to evaluating a statement having the same content as stating (5).

The only option seems to consist in a better understanding of what (5) really says given the presence of (MR) and real gaps in evaluations. Either (5) says the same as (4), namely says of itself that it has the semantic property of being false, and so we have dealt with it already, or we have

(5") This has no evaluation.

which is just false, since assuming it to be true leads to contradiction. So we may reason that either (5) has no evaluation (by being [4] in disguise) or (5) is false (by being [5"]). Saying of (5) that it has no evaluation is (5"), which is false. So if (5) *has* no evaluation we cannot say so truly. So barring inexpressibility we should say that (5) is false:

(6) (5) is false.

but if (6) is true, it entails

(7) (5) is not true.

which again has the same content as (5). Our sentence (7) about (5) not being true is, of course, not the token-reflexive (5). (7) differs in logical structure, *but not in content*.

And thus, given our reasoning about (5), (7) is false, and again we have not succeeded in saying truly of (5) that it is false. Thus we arrive at inexpressibility after all, whatever may be the semantic matter concerning (5). Of course this is disastrous, as sentences about occurring gaps should not be without evaluation themselves. Especially the claims of the gap theory should be true *simpliciter*. Semantic properties should be expressible in ordinary language.

(7) does not *assign* an evaluation to (5), and thus the *desideratum* for a theory of real gaps mentioned at the end of §2 is *fulfilled*, but even abstaining from assigning an evaluation to (5) it expresses a semantic property of (5), which *prima facie* entails facts about evaluations. The theory of real gaps thus lands in deep trouble.

§4 Opening Gaps by Pragmatics?

The Principle of Expressibility in speech act theory (cf. Searle 1969) states that everything that can be meant can also be said. It licenses the move from performing an illocutionary act (e.g. asserting “The cat is on the mat”) to expressing the illocutionary act by an illocutionary force indicator (e.g. “I assert that the cat is on the mat”). The Principle of Expressibility thus entails that there is no strong pragmatic/semantic-divide in the sense that pragmatics al-

allows us to do something with language which cannot be expressed into a sentence. Performing an illocutionary act is an intentional action, and the intention should be expressible. Reflecting which illocutionary act to perform I may deliberate on my evidence, my intentions and consult my knowledge of the felicity conditions of some speech acts. All this requires that I *represent* my intentions, beliefs and judgements (i.e. that they are expressible).

Some authors have suggested introducing or admitting a special speech act of rejection distinct from asserting the negation of a claim in question (cf. for example Bendall 1979 or Humberstone 2000).

This speech act may serve purposes of very special situations where we (i) either want to reject both p and $\neg p$ as *inappropriate* or (ii) want to express that, although for us dialetheists sometimes contradictions have to be asserted, in this case we *only* assert p and reject $\neg p$. These are special context which emphasize non-commitment or *unique* commitment. Nonetheless, in both cases we should be able to express what we want to achieve by such acts of rejection. In the non-commitment case we want to express that both p and $\neg p$ are not true [resembling the situation just discussed above] or simply evaluate both as 'indeterminate' (this being a further truth value and no real gap in evaluations). All this is expressible. In the unique commitment case the dialetheist has to make use of further negation junctors or truth operators which allow her to express the claim that the rejected sentence is false *only* (cf. Bremer 2007a). Again rejecting a claim becomes expressible, and rejection expressed in that way may be different from asserting the opposite using standard negation.

Natural language allows for a plenty of illocutionary forces, but all can be expressed into illocutionary force indicators with a clear semantics (cf. Searle/Vanderveken 1985). Thus real gaps cannot be introduced by moving from semantics to pragmatics.

§5 Universality Again

What have we seen? – Even leaving Strengthened Liars to the side and exploiting the presumably universally applicable meta-semantic resources of ordinary language coupled with the resources of a logic dealing with defaults or retractions a gap approach violates the idea that ordinary language can express all semantic facts (crucially including those the gap theory wants to talk about).

The task of universal linguistic philosophy has to be to identify the features of the universal framework of language. That framework might not be extensive, and it might be quite formal or parameter ridden what is universal in this sense, but some such framework has to be there. Therefore universal linguistic philosophy cannot restrict itself to non-universal languages. The language of such a philosophy has to be semantically closed (dealing with its own semantics), thus – at least *prima facie* – engendering semantic antinomies! Philoso-

phers more often than otherwise do not want to deal only with the structure or conditions of talking in some specific language or languages of some kind, but aims at a theory of the basic structures and conditions of having a language *in general*. This requires the corresponding logical and linguistic resources to express the universal claims. There may not be a hierarchy of languages so that we always talk in a *last* meta-language the semantic properties of which cannot be made clear, except in a further turn of the screw (a new meta-language ...). Universal theories of meaning, truth, knowledge etc. were not to have if we can talk only from some meta-language ‘down’ to some distinct object-language. But these are the very theories that philosophy is after. And notwithstanding their lip-service to hierarchy solutions of the antinomies most philosophers propose their *general* theories of meaning, truth, belief, reference, knowledge etc. They are right to do the latter, since we have such universal concepts. We can investigate and formalize the logical structures of any natural languages. That is one of the central tenets of logic and formalization. Often standard first order logic and set theory are taken as the meta-language to prove theorems about the logic in question; sometimes – as it should be in intuitionism or dialetheism – the meta-language is taken to be the same logical language as the one introduced or explained; but in all cases the logic and its formalisms are argued for in natural language texts. Natural language turns out to be *the last* meta-language, that meta-language in which the most basic formalisms of some other meta-language were introduced. And natural language turns out to be *the universal* meta-language in that all the formal constructions and sentences of some new system can be translated (read) as ordinary sentences with some formal regimentation. There is no extraordinary special or deviant new logic which can say something that we cannot say in (some) natural language. Natural language thus contains the capacity to interpret all these systems. Therefore the issue of gaps is most pressing concerning natural language. Whatever can be reasoned about gaps can be expressed in natural language. What structures are responsible for this may be the task of advanced philosophy to find out. Without semantic closure we would not be *able* to elucidate a concept that we seem to have! Corresponding to this universal scope of its investigations this philosophy needs the logical means to speak universally. It needs a universal (and thus paraconsistent) logic. A truly universal (paraconsistent) logic can be employed everywhere, supposedly containing a way to distinguish consistent from inconsistent contexts, without loss of proper logical power in comparison to standard logic (like in Adaptive Logics).

Dialetheism can be qualified in that context in two ways: (i) the primary concern is semantic closure and not the assurance that there are true contradictions, i.e. if we can have semantic closure and universality without contradictions we are even the better off, as there is nothing inherently valuable about true contradictions, they are a nuisance the dialetheist is ready to accept only because we cannot give up on semantic closure and universality, (ii) the con-

traditions accepted pertain to our meta-representational skills (i.e. they affect our concepts of truth, denotation or concept application) and result in contradictory evaluations of claims *about* representations, there are no true contradictions with respect to (first-order) physical properties of space-time regions.

§6 No Gödelian Gaps Either

Gödel's famous *Incompleteness Theorems* show the presence of gaps of another kind: gaps of negation incompleteness. [A formal system S being negation incomplete in case neither $\vdash_S A$ nor $\vdash_S (\neg A)$ for some sentence A .]

Priest (1987) uses the Gödel sentence as another argument for dialetheism: supposing *Church's Thesis* the informal reasoning that the Gödel sentence G is true can be captured by a formal system N for naïve proofs. Sentence G for some logical system expressive enough says of itself that it is not provable in that very system. Sentence G for the system N is provable in N (as exhibited by the informal argument usually given for its truth), and thus negation incompleteness is avoided, at the cost of inconsistency, as system N meets the conditions not only to express and represent the concept of provability, but to express – and even represent – the concept of truth as well. Dialetheism accepting inconsistency in any case (dealing thus with the semantic antinomies) can reap the benefit of avoiding negation incompleteness. For Priest, thus, Gödel's argument shows a dilemma: either we achieve negation completeness or consistency. And, as we can reason to the truth of G , achieving negation completeness has to be our priority.

But isn't the contradiction engendered by G too much even for the dialetheist? Having G means having a provable sentence claiming truly its own non-provability, and because G is provable $\neg G$ is true, thus it should be provable as well, as we can informally reason to its truth.

As we want the paraconsistent equivalent to soundness N should not prove sentences that are false only. [Paraconsistent system may prove sentences which are true and false at the same time, but proving a sentence that is only false (i.e. not true at the same time) would be a hypercontradiction putting the system used into doubt (cf. Bremer 2005a).]

Consider the following argument:

G should not be false only, as it should then be provable given convention (T) for truth

$$(T) \quad \text{True}(\text{"p"}) \equiv p$$

from right to left (so-called 'T-in') and contraposition. If G is at least true, it is not provable, given (T) from left to right (so-called 'T-out'). As we can informally reason to G 's truth, this reasoning is captured by a proof in N , N thus proving G . So we have

$$(1) \quad \vdash_N (\exists x)\text{Proof}(x,G)$$

By paraconsistent soundness and T-out we thus get

$$(2) \quad \vdash_N \neg(\exists x)\text{Proof}(x,G)$$

This means that there is some number m which both codes a proof of G , by (1), and does not, by (2)! This is too strange as ‘Proof’ is a primitive recursive, algorithmic and deterministic relation. So could we ever have (1) and (2) together?

End of argument.

Without mystery we cannot give up *Church’s Thesis* and the existence of N . Representing the presence of a proof by ‘Proof’ should be available.

One measure might be a paraconsistent account of algorithms: one and the same deterministic algorithm providing different answers or an inconsistent answer on the same input. This is a *desideratum*, however. And it seems to clash with our intuitive notion of algorithm [see Appendix to this chapter].

It is no option either to stress realism again (claiming provability to be different from truth) as G is available and provably *true* in N .

One may claim that m makes (1) true by really coding the proof of G , and (2) is thereby another provable sentence of N resulting in saying of m falsely that it does not code the proof of G . Thus m turns out to be an inconsistent object in N . And (2) is another contradiction, its truth being guaranteed by the provability of G . As there are other inconsistent objects in N already, this may not worry a dialetheist. The dialetheist may worry as coding a proof is a primitive recursive relation. So that saying of m that it does not code a proof of G contradicts the result of a respective algorithm. The algorithm, however, will in no way be suppressed in its workings: it delivers that m codes a proof of G , making (1) true. (2) just falsely contradicts this positive output. It cannot interrupt the algorithm which properly establishes (1).

A dialetheist can, therefore, embrace the truth of G , and so avoid Gödelian gaps of negation incompleteness.

Appendix

What is an Algorithm?

This appendix provides a definition of ‘algorithm’, as the concept will again be used in chapter 5.

The fundamental concept in the theory of computation (and thus in the theory of representations) is that of an algorithm. Some procedures are said to be ‘algorithmic’, some problems have an ‘algorithmic’ solution, and many others have not.

Let us approach the definition of an algorithm by a route that highlights the need for some distinctions the absence of which has caused a lot of confusion about something being ‘algorithmic’ (and especially confusion in claims of refuting *Church’s Thesis*).

Suppose a TM M_1 computes the decimal expansion of Π . Should we say that Π is computable?

In a *narrow* sense Π is not computable as we never compute Π (in its entirety). We only compute Π up to some digit. Computable in the narrow sense is the n^{th} digit in the decimal expansion of Π . This computation terminates. One may thus say that Π is computable (in principle).

[Turing’s classic paper (Turing 1936) deals with irrationals thus computable!]

This gives us a notion of algorithm in the *narrow* sense.

An *algorithm* is:

- (i) *Implementation neutral* (abstract)
- (ii) *Effective* in its individual steps (they do not require ingenuity)
- (iii) *Finite* (given finite input and a finite number of machine states in a finite time the algorithm terminates with a finite output in finite space if it terminates)

We see implementation neutrality in one algorithm being carried out by devices that may differ in their architecture and material. If ‘program’ is understood as ‘program in one or the other programming language’ then different programming languages provide different ways to implement one and the same algorithm.

We see effectiveness in the simple basic steps of the different paradigms of computability (e.g. moving the head of a Turing Machine or reading a register in a register/abacus machine). The individual steps are basic mechanical operations.

Partial recursive functions are computable in the narrow sense. [Partial recursive functions correspond to Turing Machines, *inter alia* (cf. Boolos/Jeffrey 1989).]

If one strengthens the finitude constraint to

(iii)' *Finite* (given finite input and a finite number of machine states the algorithm terminates in a finite time with a finite output in finite space)

partial recursive functions are computable in that narrowest sense as well in as much as there is always an extensionally equivalent terminating total recursive function.

In a *broad* sense Π may be said to be computable itself as M_1 computes it. M_1 is algorithmic in a broad sense as it is an infinite sequence of terminating sub-computations.

This gives a notion of an algorithm in the *broad* sense.

An *algorithm* is:

- (i) *Implementation neutral*
- (ii) *Effective*
- (iii) *Finite* in its sub-computations (each of which given finite input and a finite number of machine states in a finite time terminates with a finite output in finite space)

Halting algorithms then are those that enter after finitely many of such sub-computations in a halting/acceptance state. Non-halting algorithms either stay in a non-halting state or consist of an infinite sequence of sub-computations.

Allowing for unbounded sub-computations violates the finitude constraint. In this way – *inter alia* – the *Halting Problem* becomes solvable.

If some notional computing device is (said to be) 'hyper-computational' this means that it can solve problems not solvable by Turing Machines. This capacity depends (in the notional machines proposed so far) on: infinitely many states, infinite input, infinite time, space or infinite precision of measurements – all features beyond the intuitive concept of computation and algorithm which involves *finitude*!

A special case are Oracle Turing Machines, which resort to *ineffective* steps of computation (by consulting the oracle). All these notional machines defy realization. None of them refutes *Church's Thesis* or even the *Physical Church-Turing-Thesis* (that a machine beyond the Turing Limit cannot be realized).

Conceptual Analysis in Ordinary Language Philosophy Revisited

Ordinary Language Philosophy (OLP) has become unfashionable with the rise of 'naturalism' and the cognitive science approach to traditional philosophical issues. There are some hints (e.g. several recent books) that with meta-philosophical reflection some reconsideration of OLP takes place, to the advantage of Analytic Philosophy. Philosophical fashions are not more sustainable than other fashions, so that ideally the merits of supposedly 'superseded' approaches should be incorporated into their descendants, notwithstanding disagreements with individual claims made by ordinary language philosophers – where in philosophy do we not disagree with parts of theses or do not recognize partial error? OLP with its focus on the central importance of language and the impact of established usage, admitting language authority in philosophical debates, contains valuable insights and methodology for any future Analytic Philosophy.

OLP, of course, need not and should not be taken as the only form of philosophy and philosophical reasoning. Obviously the (following) arguments in favour of OLP are not simply exercises of OLP, and other philosophical approaches (most notably in the context of the tradition of Analytic Philosophy its so-called “Ideal Language Philosophy”-branch [ILP]) have their merits as well.

The present chapter does not aim at justifying OLP as the best philosophical approach, not even within the confines of analytic philosophy, as should be clear from the rest of the book [see also the last part of the paper and the outline of my understanding of analytic philosophy and the (limited) role of OLP therein in (Bremer 2005, pp. 390-94)]. The present chapter rather aims at justifying OLP as an integral part of analytic philosophy, not to be completely rejected or neglected.

5.1 General Discussion

§1 *Prima Facie* Worries about Ordinary Usage

Gareth Hallet (2008) organizes the issue of 'linguistic philosophy' as centring on the 'authority of language', a well-chosen point of departure.

As we like to deal with *philosophical* issues we often want to supersede a more chronicle of usage. To do otherwise rests authority on (all) philosophical issues on a mere socio-historic record, it seems. Where should the philosophical qualification of that usage come from? Usage has become embedded over time in (useful) ways of acting, but may have had a limited scope of situations confronted. Thus usage stays silent on many problematic scenarios. It just does not decide one way or the other on how to employ an expression in these circumstances. It is not fixed in universally applicable criteria of sufficient and necessary features of something. Thought experiments, thus, cannot be decided, at least often, in favour of one of the supposed views based on them. Intuitions with respect to them are not completely grounded in language then, but contain minimal theories stemming from one's prior view on the issue in question or related affairs.

Why should we expect ordinary language to have sufficient authority in cases of theoretical issues? The occasions of such questions being spoken about may be severely limited. Ordinary language seems to be the wrong place to look for (hidden) theories and well-defined concepts.

§2 Philosophical Issues and Ordinary Usage

One may consider philosophical issues to be an exception to these worries. As many of them concern foundational issues in our conceptual scheme one may surmise that (even) ordinary language contains enough structure and rules concerning them. One may even insist that in case that ordinary usage does not reveal something about them then nothing (else) can be revealed about them. Anything important about foundational concepts has to have left its trace in ordinary language, otherwise these concepts just would not be foundational but optional.

So, looking at, listening to ordinary usage on foundational concepts is a live option and may even delimit where we cross from conceptual knowledge to our additional intuitions stemming from other parts of our world view or our cherished theories. We may discover, to our dismay, that some of these foundational concepts are not sharp in the sense of laid tracks of sufficient and necessary conditions of applying a term. This in itself may be a discovery worthwhile. Not all conceptual links have to be drawn by (complete) definitions.

We may see the proper role of a foundational concept despite its vague nature. Some questions have not arisen in human history and may not be live concerns

to be prepared for (except in some option of coherent extension of the concept to new situations of usage).

Therapeutic concerns may also enter at this point, as one might point out where a (philosophical) use leaves the boundaries of ordinary usage without being explicit of its stipulative character. The therapy points out what content does not properly belong to the concept in question. Therapy of philosophical extensions of concepts is linked to the idea of sufficient foundations: philosophers sometimes want to regulate affairs which need no regulation as there are hardly occasions on which they may arise. Concepts may be suited to ordinary circumstances, and here are sufficiently regulated. Any supposed conceptual gap or missing clarity should first be considered as to its likelihood of playing any role in ordinary affairs at all.

Again, affairs being side-lined now may become important tomorrow, but unless then they are just artificial. Stipulation may deal with them as real occasions arise.

§3 Ordinary Usage Is Not Theoretical

Ordinary usage, although non-arbitrary, does not express a considered theory which guides it. Common usage should not be identified with 'common sense' or folk theories, simply because huge theoretical differences can be expressed within the same language. The distinction between language and theory is partially well drawn as not all theoretical assumptions enter into meaning and change the way expressions are used, a distinction sometimes highlighted by separating analytic links (analyticity) in meaning from entrenched or not so entrenched opinions and theories in which an expression occurs. The distinction is partially not well drawn as language use *may* involve minimal theories, but usage primarily concerns concepts (*via* the way words expressing them are used) not theories. Concepts may sometimes be defined in a way corresponding to theories, more often they are left vague or just referential (in the sense that the corresponding terms are rigidly designating kinds of entities). In most cases usage can be a guide to identify the natural or non-natural kind of entities linked to an expression in question, or to capture the core assumptions linked to a (vague) concept. Such core assumptions, although they are theoretical in involving beliefs about the world, can be non-theoretical (i.e. lower level) with respect to a theoretical dispute under discussion. Either, in the simplest case, by dealing with what counts for a theoretical dispute in question as 'observational' or – in more interesting cases – containing the agreed upon conceptual core shared between the alternative theories.

§4 Intuitions and Linguistic Judgements

OLP does not rest its claims on 'intuitions'. "Intuition" has too many different meanings: these range from opinions based on 'common sense' or folk theories

(so that claiming something to be 'intuitively' so may hedge the claim made) to the intellectual apprehension of conceptual insight (e.g. in some philosophies of mathematics). One might call reliance on one's unsystematic (i.e. pre-theoretical) understanding of language rules 'intuitive', but as the term has been used for quite different forms of belief, it may be better to speak of 'an expression of language competence' or 'linguistic judgement' or '(pre-theoretical) knowledge of language' or 'meta-linguistic beliefs'. In OLP we investigate possible cases under the directive of *what we would say if*. The evidence which turns up with this rests on linguistic judgements or pre-theoretical knowledge of language. [Having said this we may sometimes follow the custom of calling such judgements "intuitive", but only if the context does not allow to confuse them with common prejudices or folk theories.]

Philosophical 'intuitions' either appeal to shared convictions in some (folk) theory, which carries little argumentative weight, or are guided by a (partial) apprehension of rules of word usage. Intuitions in this latter sense are philosophically useful and necessary as an element of analysis. Positively one may surmise that the new fashionable recourse to 'intuitions' stems from a dissatisfaction with 'naturalism' and mere stipulation of language forms. Pleas to 'intuitions' want to come close to a source of *philosophical* insight, where this source better be language.

In many cases it may be difficult to separate linguistic judgements from folk theories. Linguistic controversies show that competent speakers may err in explicating the constitutive rules of their language. Their linguistic judgements are often superseded by their (folk) theories. A mayor difficulty in analysis consists in removing accompanying views and attitudes from judgements of linguistic conformity. Tacit semantic knowledge thus cannot easily be turned operational in identifying conceptual links or analytic sentences.

Can an ordinary language philosopher just rely on his or her own linguistic judgement? Critics of OLP often complain that the observation made – in the proverbial easy chair – are too unsystematic and merely proto-science awaiting a more methodological and representative survey done in linguistics. This criticism suggests that analysis of meaning needs empirically scientific methods of observing language use. Interesting as such linguistic surveys may be, one can nonetheless reject this objection to OLP. The (tacit) knowledge of language that OLP tries to capture and analyse has to rest in *each individual's* competence of language use, including the philosopher. As speakers of a natural language we might err in our conceptions of usage and word meaning, this being the reason ordinary language philosophers putting their theses to the criticism of their audience, them being competent speakers as well, but as competent speakers we know (tacitly at least) what we need to know and are not in need of statistical data on language use (cf. Hallett 2008, pp.153-54; Hanfling 2000, pp.53-56). This proves right even more so when considering fundamental concepts. The ordinary language philosopher participates in the

conventions of her linguistic community. Aiming at her own (tacit) grasp and attunement to these patterns of usage she aims at the (tacit) grasp of them of any competent speaker. She does not infer to the others' knowledge by means of an syllogism (involving *assumptions* with respect to their linguistic community) or statistical data. *Prima facie* and by default the ordinary language philosopher possesses as much knowledge as is targeted by a conceptual investigation.

§5 Conceptual Analysis Is Not Empirical

Reports of statements or assertions about language or use report events, i.e. are empirical sentences. A description of a pattern of usage in a linguistic community is an empirical sentence. Neither of these classifications implies that conceptual analysis is empirical. A description of a pattern of usage describing the rules or conventions of a linguistic community is true only if these rules or conventions are *in force*. Reporting rules or conventions does not transform them into reports. The reports are descriptions backed up in regularities of behaviour (i.e. events), described empirically. The regularities in question exist because the speakers of the linguistic community orient themselves (at least tacitly) on rules or conventions, which are norms and expressed by the use of deontic modal vocabulary. Assertions made by oneself or witnesses about language are events, but their content are judgements whether a linguistic rule has been applied correctly or not; rejecting, for example, a sentence as a category mistake contains the judgement that some semantic rule has been violated. The semantic rules in question concern conceptual links (like 'numbers are not spatio-temporal'), which at least partially constitute the concepts involved; completely so only if a concept can be completely analysed into a definition involving informative necessary and sufficient conditions of applying the concept, which may be feasible only for a small minority of concepts. Even if most concepts, however, are atomistic in the sense of not having such a definition (as claimed by Conceptual Atomism [Fodor 1998]) they are accompanied by conceptual links: knowing or possessing them within the framework of our concepts involves knowing of these conceptual links (cf. Bremer 2008, pp. 31-45). Sentences expressing conceptual links are analytic and thus in the traditional sense *a priori*. Such sentences may be embedded in deontic modalities so that rules result, which demand that the conceptual links have to be taken into account, that words are only employed assertively in a way that does not result in nonsense (i.e. that the constraints of the conceptual links are obeyed). The rules may be understood as more specific (e.g. 'In assertions never apply a predicate implying spatio-temporal existence to a singular term when the singular term, say a numeral, is used to designate a number') or there may be a few general rules (e.g. 'Assertoric use of a sentence should not contradict the analytic sentences'). The latter alternative has the advantage that we represent our linguistic knowledge more efficiently: we have the rules of reference and

meaning postulates, and what follows only by them expresses conceptual links. We need these representations in inferring anyway. Rules demand in the general fashion indicated semantic correctness. Individual rules and verdicts on use follow from the combination of the two components. Because of the 'authority of language' [cf. §7], sentences analytically true (true by conceptual links) nevertheless can safely be assumed to speak truly about the reference of the words employed: "Cats are animals" is analytically true: even if "cat" cannot be completely analysed into necessary and sufficient conditions of being a cat – apart from reference to an usually unobservable genetic code – a partial definition of "cat" consists in this postulate. Nevertheless the sentence tells us that cats *are* animals. The sentence is *about* cats, not about concepts. *That* the sentence is analytic tells us something about the concepts involved, thus our conceptual framework and our language.

The results of conceptual analysis thus contain several kinds of sentences or qualities of sentences ('analytic', 'empirical'...), the fundamental sentences, however, are those expressing, re-constructing or explicating conceptual links, i.e. non-empirical sentences. Again, reporting that we *have* such and such a conceptual framework, which is an empirical anthropological claim, does not make philosophical re-constructions of the structure of that system empirical claims. Claiming that most mathematicians believe that Peano Arithmetic is true and thus use the system is an empirical report, that 0 has no predecessor is not.

§6 Contextualism

Analysis of language does not focus on individual words, not even individual sentences, but on the use made of them in some contexts. A linguistically therapeutic remark in most cases does not put into question a sentence in general, but a inappropriate use of it, or more restrictedly, an improper understanding of a usage of a sentence. There lay many of the errors and aberrations analysis of proper usage reveals. The mistake often rests in assimilating in our understanding of them sentences properly used on one context to similar (or even the same) sentences improperly used – or at least improperly understood – in other contexts. Few will look for an entity corresponding to the subject of the sentence "It rains", but talk of discrete objects misleads many in assimilating sentences like "Our will is indefatigable" to this, postulating the common will as an entity.

OLP, however, should *not* be identified with a 'meaning is use'-theory of meaning. Analysis of usage includes analysis of pragmatic features of usage, as covered in a theory of speech acts and illocution (as introduced by the ordinary language philosophers Austin and Searle). Otherwise Grice's theory of conversational implicature or Relevance Theory in present cognitive science will be mistaken for refutations of OLP in general, whereas Grice himself establishes his theory by focussing on the varieties of usage. Although quite a

few individual claims (e.g. by Wittgenstein on 'know' or Malcolm on 'dreaming') may lack from insufficiently distinguishing semantic and pragmatic factors, concluding from the absence of intelligible uses of an expression to its meaninglessness, a 'meaning is use'-theory is not constitutive for OLP.

§7 The Authority of Language

Where does the authority of ordinary language come from? The question is misleading as one may read it as presupposing a trust in common sense, which often went wrong and cannot claim scientific authority. The authority of ordinary usage must not be confused with a privileged role of folk theories or folk interpretations accompanying this usage. Such folk theories or interpretations may illicitly move from the contexts of ordinary usage to the context of (scientifically) theorizing about the world, and in that context scientific theories usually fare better.

Of course ordinary usage has neither authority in a verbatim reading of it in clashes with scientific discoveries (the sun just does not "rise"), nor does it exclude introducing more appropriate ways of speech for scientific purposes. There is no authority of a supposed general common way/context of talking over some specific context of language use. The authority resides, firstly, within contexts of usage. Established patterns of usage rest on a history of successfully employed language. Such patterns fit to reality and human endeavours in it. Therefore they also are often descriptively adequate (enough). A usage following these patterns thus possesses a higher chance of being successfully embedded in our dealing with the world, including its description (starting from simple cases of following the usage of "tree" to identify trees). Very often the use of a sentence in a situation corresponds to the world because it corresponds to established usage. Such *correspondence of usage* (intersubjective *coherent* usage) precedes correspondence of language and world (e.g. in the sense of a robust correspondence theory in which true statements correspond to facts). This role of correspondence of usage founds the authority of ordinary usage.

Analytic sentences mirror the authority of language. "All cats are mammals" tells us that all cats are mammals (i.e. it tells us something about reality). One of the major flaws of Logical Positivism – and thus of much ILP – rested in the thesis that analytic sentences are empty of content or 'tautological'. Definitions and analytic sentences embody substantial insights and beliefs about reality. If 'cats' are not mammals (i.e. the objects we use to talk about with the word "cat" and refer to with the concept expressed), one may rigorously claim that there are no 'cats' given our way of speaking, but the next step will be to drop that analytic sentence and so far inappropriate definitions involved. Definitions cannot be wrong if one considers them as rules, but they can be superseded by better definitions. By definitions and analytic sentences we express our recognition of the nature of the properties involved. Therefore derived or

discovered analytic sentences can provide *substantial insights* with respect to reality, not just with respect to language or concepts. Seeing philosophy as being concerned with analytic sentences thus does not remove it from being concerned with reality (as also stressed by Williamson [2007] despite his criticism of the linguistic turn and his – well placed – criticism of epistemic understandings of analyticity). Analytic connections correspond to metaphysical connections (i.e. connections in the nature of the properties involved, even stable across varying natural laws).

Truth-conditional semantics coupled with the meta-rule to assert sentences according to their truth conditions tries to capture this double correspondence between language and reality.

OLP should not be equated with anti-realism tout court. At least some realist theories of meaning combine their realism with a focus on norms of usage and justificational procedures close to the meaning of a sentence, where they take the core of the meaning of an expression to rest in the *referential* links between the concept expressed by the expression in question and parts of reality (e.g. objects, events and their properties), so that proper usage is (at least indirectly) a way of tracking features of reality (cf. Bremer 2008, Peacocke 2008). Therein rests the authority of proper usage.

§8 Conceptions of Concepts

Philosophical theories may often be just such elaborated folk interpretations of ordinary usage that remove it out of its ordinary context or mix different contexts of usage (e.g. reading expressive utterances as referring to entities just as descriptions do, thus arriving at an extravagant ontology). One may dismiss such 'theories' out of hand by outlining their deviant usage. One may also criticize them. Criticizing such 'theories' may take two forms: (i) rejecting them by confronting them with better scientific theories, or, more interesting, (ii) rejecting them as theories, but, at least in some cases, maintaining or even elaborating them as an insight into our naïve conceptual framework, or into some common mistakes invited by our conceptual framework. OLP need not claim that in our conceptual framework all discoveries of future science are hidden. Our conceptual framework *may* contain constitutive errors or misleading constructions. Nevertheless and even because of this OLP puts emphasis on revealing them as they are.

In analysis of ordinary usage we understand our concepts at a level even beneath folk theories. Still, what we analyse here are the basic *conceptions* of our concepts, of our conceptual framework. These conceptions need not be entirely accurate. That they are not could be seen either by relating them to other investigations about our conceptual framework (as of theories of cognition or linguistics) or because of incoherencies in ordinary usage. Only if ordinary usage was acceptable as it is in its patterns, ordinary usage would carry *full authority* on our basic conceptions of our conceptual framework. Incoherence,

however, clashes with the idea that even what is said about vague concepts need not be vague, that what is said about, seemingly, inconsistent employment of a concept need not be inconsistent itself. A supposedly incoherent usage at least invites further conceptual distinctions which then distribute the apparent clashes over at least two sub-concepts each of which with a coherent usage.

One method to keep folk theories at bay may be (following Austin) to attack a concept not from its disquotational linguistic expression (e.g. analysing how we use “freedom” to analyse the concept of freedom), but from accompanying, still central terms which are not as closely associated with folk theories (e.g. analysing “involuntary” and “could have done”).

If we just report usage this type of descriptivism ascribes full authority to usage and *records* a state of our conceptions of our concepts. Getting to know this state possesses some interest in itself, and might be considered as part of our cultural history. Of course often philosophy aspires to more than to a history of ideas, which then makes it move beyond mere descriptivism. On the other hand usage changes and thus even the core level of our conceptions of our concepts may shift, in almost glacial speed, with it. Supposedly it shifts given new contexts of usage as culture and technology change, new words entering the language, and even moving away from perceived – however vaguely so perceived – misconceptions. Could we pinpoint a state of usage we might even relate that period's usage to that period's most fashionable theories, including the wrong ones. This again would be part of a more comprehensive cultural history.

In case conceptual analysis tells us that some fashionable identification (say of mind and brain) cannot be stated save conceptual confusions this does not tell us that mind and brain are not identical, and it does not tell us that neurophilosophy has to stop. It tells us, however, that our concepts cannot be unified so easily. It may at least cast into doubt any attempt at such identification as we have our concepts not by accident but as part of our more or the less successful cognitive equipment. It points at least to the need of a kind of conceptual revolution. And in case of a nonsensical claim of identification of mind and brain it may not be the mental vocabulary that has to give: chemistry and physics could only be united after a conceptual revolution in physics; in analogy a conceptual revolution in the neurosciences and physiology might be needed to support any aspirations of an identity theory.

Further on, attesting some conflicts in our conceptual framework may *not* find an easy remedy – thinking of it like changing a flat tyre! Especially so if some substantial part of our conceptual framework (say basic psychological or metric or semantic concepts) is innate. We cannot drop innate concepts at will. We can only work around some of our conceptual limitations if that was necessary. Compare: However much you liked it, you cannot tell your bladder to produce whiskey. You cannot see some perceptual illusions in the 'proper' way.

In just this fashion you might not be able to drop the naïve concept of universal truth and naïve set comprehension. Any call for conceptual revolution should heed (i) to the at least presently given limits of human cognitive nature, and (ii) to the advice that you should not drop what works in most situations and what one cannot improve at will, so that what one ends up with might be some form of conceptual reform process in which extensions of usage or extensions of our conceptual equipment work around the diagnosed errors or limitations. Having abstractly outlined such vague possibilities does not mean that there ever will be pressing philosophical reasons to proceed in this this. (Even neuophilosophy may turn out a blind alley after all.)

§9 Improving on Usage

As usage changes itself to adapt to new circumstances, we may *intervene* to foster its better adaptation or coherence. Introducing finer conceptual distinctions and excluding some ways of word use are ways to regularize usage anew. Analysis of usage so precedes new regulation. Philosophical analysis leads to a *normative* activity of tweaking rules of usage, of upgrading our conceptions of our concepts.

In analysis, in general, we can see the constitutive elements and so gain understanding, even if we leave them as they are, put things together again, in synthesis, as we found them before analysis. We can, however, as well synthesize them in an improved fashion so that synthesis is not just the reverse of analysis, but also an attempt at practical advancement. This applies to technical devices as well as to conceptual frameworks. Creative synthesis achieves a reconstruction of a concept. Carnap, at least sometimes, pursued this approach as 'conceptual explication'. Artificial languages can serve to highlight *some* structures and functions of ordinary language, insofar they are *one means* of analysis.

When we distinguish philosophy from the sciences by conceiving it as a *meta-science* not concerned with the world at large, but our concepts and conceptual frameworks of it, this does not mean that philosophy is uninterested in a better fit of our conceptions to the patterns of the world or just to our concepts themselves. Philosophy as an *activity* (which may occur outside of department boundaries) distinguishes itself by reflecting, analysing and re-regulating language and its conventions, exhibited in usage.

With respect to words foundational in special sciences this philosophical activity may involve more re-regulating as scientific theories change faster than our general opinions and attitudes towards the world and ourselves. At this stage, say in case one deals with cognitive science, considerations of a wide reflective equilibrium take centre stage.

With respect to words considered belonging to or relevant to pure philosophy itself (like "knowledge" or "free will") this activity may proceed much more cautiously to avoid the trap of stipulating present day opinions and theories to

be conceptually ingrained. All motivation and evidence for improved usage has to come from criteria of improving coherence in present usage. Missing a clear account of such improved coherence, supposed incoherencies have to be left in place and mapped as part of our conceptual landscape. Philosophy leaves then everything as it is (in usage) and endorses the authority of usage on an elucidation of our fundamental concepts. The benefit of even this strongly descriptive enterprise rests in increased understanding by analysis, even if synthesis is not creative, and in the therapeutic use of rejecting some (philosophical) theories as clashing with proper usage, which as part of our life carries more weight than those deviant views. A conception Wittgenstein at least sometimes pursues.

Explication in Carnap's sense and the activity of creative synthesis or reconstruction of usage in the light of scientific purposes are meta-scientific activities, as such philosophical, nonetheless continuous with foundational reflection in individual sciences. OLP sets itself more apart from the sciences and guarantees philosophy its own field and status. Even if formal reconstructions are employed within analyses their purpose is not to alter usage or to stipulate new language forms. Therefore the distinction between ordinary language philosophy and ideal language philosophy should not be understood as excluding formal methods from OLP, but still serves some purpose in stressing the non-instrumental perspective that OLP, in contrast to huge parts of ILP, has on language. Explication and creative synthesis have to answer the criticism that they just sweep the complexities of an issue under the carpet of a redefined expression, that their redefinition may be nothing more than confusion about the real issues. Many formal explications just seem to change the subject as the original issue was just too intricate to be dealt with in that fashion.

There can be an explicative, re-constructive approach *not* tied to empirical sciences, namely in case we recognize, supposedly with the help of OLP, that our conceptual framework contains misconceptions and epistemology tries to see more clearly or even tries to intervene into the future development of our conceptions. Conceptual re-construction and re-regulating involves the construction of models and (partial) languages. It combines the traditions of OLP and the study of formal systems and languages.

Descriptivism, in contrast, seems to be a very conservative approach. It need not neglect the change of usage, but sits back and leaves present day challenges to our concepts catch up with the slowly changing usage.

Sometimes inventing technical jargon helps to make fine-grained distinctions, explications may aim at a proper updated definition of a term which is more perspicuous than the former one. Often, however, re-definitions are employed not to capture the full control of a term but to facilitate the development of one's cherished theory, disguised as dealing with the old subject matter. One might regard it as ironic that those positivists or 'naturalists' who insist on be-

ing concerned not with verbal disputes but the matters themselves, seem to believe that by stipulating a new definition of a term the old problems related to its subject matter (reference) are solved. These problems are rather shoved under the carpet by trying to make them inexpressible. Re-definition and a move to newly regimented (formal) languages have therefore to answer the suspicion that the gain in rigour is outweighed by the preceding flight from the complexities of ordinary language.

§10 Ordinary Language in General

OLP aims at *concepts*, conceptual distinctions and conceptual links. It is only instrumentally, methodologically concerned with an individual language. OLP is neither lexicography nor historical linguistics. The discoveries to be made pertain to all human languages in as far as they are all able to express our conceptual system, even if some language may employ more words to trace fine-grained distinctions. The examples discussed by English ordinary language philosophers in Oxford can be transferred to examples in German discussed in Bielefeld. The writings of OLP have been translated into many languages.

Even though usage may shift the underlying conceptual distinctions and links can at least be captured by rephrasing what is or was said in another language, and what was (considered to be) conceptually true (or false) has to be (considered to be) conceptual true (or false) as expressed in some other language as well. Even grammatical idiosyncrasies of one language which have no direct correlate in another language may point to conceptual issues worth to be expressed by circumscribing them in some fashion in another language without these grammatical idiosyncrasies.

All this applies the more OLP looks at foundational concepts shared in all human civilizations.

Some adherents of conceptual analysis want to detach conceptual analysis from analysis of (ordinary) language, and see it concerned with our conceptual judgements and not with language at all (e.g. McGinn 2012). This may rest on a confusion about the role of language in linguistic analysis: language need not be taken by linguistic philosophers as the primary *object* of philosophy, but analysis of language (usage) is taken as one crucial or even the privileged *method* of getting at concepts. “Definition” applies to words at least as well as to concepts. Proposing a definition and testing it with cases (similar to proposing a hypothesis and testing it) explores whether the definition covers all cases by testing our judgements to the applicability of a term (the meaning of which contains the concept referring to the property ultimately under investigation). If language was not methodological essential one has (1) to account in some other way for the shared possession of concepts, which on the other hand every theory of concepts has to do, and (2) to find some other way to identify a concept in question intersubjectively. Methodologically language helps to identify a concept in question as the core of the meaning of an expression em-

ployed. Further on, many concepts (especially those for social institutions) depend on language and rule governed communities. Concepts like that of 'marriage' cannot be separated from special speech acts that *constitute* respective social facts. Many if not most of the concepts interesting philosophers will be of this kind. Epistemological concepts like that of 'knowledge' are at least indirectly tied to language (e.g. by the link from feasible assertions to know something to justifying them towards an audience, of course using language and appealing to shared conventions). Moreover, one may argue that concepts involving powers of reflection and self-representation in thought need language ('inner speech') as representational device. This applies to all forms of shared knowledge ('common knowledge') essential for conventions, and arguably to a full-fledged concept of 'belief', as this involves reflecting on one's beliefs and their interrelations, and their relation to the world.

So, although conceptual analysis – by definition – aims at concepts, the privileged method to do so is linguistic analysis. As conceptual analysis aims at concepts shared between individual natural languages, no individual natural language is essential for it, and all its cases of analysis have to be transferable in principle from one language to another.

5.2 A Short Overview of Methods in Ordinary Language Philosophy

§11 The Paradigmatic Methods of Ordinary Language Philosophy

The methodology of OLP developed over the years (cf. Hoche/Strube 1985, von Savigny 1969). It comprises – besides the usual methods of evaluating hypotheses by, for example, comparing their explanatory power – at least the following:

1. *Substitution tests* within sentence frames (as practised e.g. by Ryle to distinguish 'categories' and applied by Sibley to analyse aesthetic judgements) or substitution tests in general discourse, including whole utterances (as practised e.g. by Strawson in stressing performative aspects of “is true”). Categories are not only *part* of an analysis (e.g. classifying something as 'abstract'), but elucidating *them* engages in analyses of kinds (be it ontological, like 'abstract', or epistemic, like 'synthetic').
2. *Paradigm case arguments* to outline the core use of an expression, and to identify against this the deviant use of an expression (as practised e.g. by Moore with respect to “good”). [see §12]
3. *Situating sentence use within a context of linguistically mediated action* or dialogue (as practised e.g. by Wittgenstein with imagined 'language games' and simulated exchanges, often with Wittgenstein's *alter ego* as interlocutor).
4. The preceding method may include *imagining situations* and asking *what we would say* in such circumstances, thus distinguishing empirical regularities of usage from conceptual rules which support counterfactuals (as practised e.g. by Austin to differentiate felicity conditions and illocutionary forces). Even *thought experiments* may outline the borders of usages, and may then introduce a sharpening of our conceptual boundaries, as extensions of an established usage. Such an extended usage in this case does not go against established use or the authority of language otherwise present. With imaginary situations OLP is more exploratory than with the more descriptive approach to ordinary language in ordinary situations. With imaginary situations OLP asks speakers (in real life or in supposition) what they would say. Conceptual boundaries are explored as well as the borders between the conceptual (i.e. essential usage) and the factual (i.e. possibly extended usage). With respect to possible situations those which are physical and maybe even technologically possible test for our linguistic judgements in other situations, but still within the boundaries of our culture. Strawson employs thought experiments of this kind in *In-*

dividuals to delineate the inevitable features of our conceptual framework. Far-fetched possibilities, however, can turn out to be problematic easily. Generally ordinary usage *need not* cover situations which cannot arise, so our judgements or lack of them with respect to such situations should carry not so much weight. Moreover, such 'thought experiments' are often framed in the light of the judgements the author wants to endorse. We meet bizarre stipulations or incomplete descriptions which challenge the claim that such situations are possible at all. A description of such a situation by itself does not prove its consistency. We neither have a complete list of (semantic) postulates which have to be satisfied, nor do we possess a general procedure to determine consistency (for meta-logical reasons, at least). So one reaction to a supposed situation of application or 'thought experiment' may very well consist in rejecting the case as 'not possible', 'not conceivable' or in arguing for its inconsistency (maybe in its own terms or in terms of the conceptual links supposedly to be tested). Thought experiments should be treated with care!

5. *Semantic-pragmatic combination tests* to distinguish semantic entailment from pragmatic (illocutionary) implication (as practised e.g. by Hoche with respect to the concept of moral obligation). These tests can cope with the problem whether judgements on usage are founded in knowledge of meaning of the sentences overtly used or in knowledge of illocutionary acts or other pragmatic conventions. For instance "I am speaking right now" is a pragmatic tautology, in contrast to "I am speaking, so there exists something (speaking)", because the latter is an instance of existential generalization (thus semantically valid) whereas the former, although true while being uttered need not be true, as shown by the consistency of the combination "It is possible that I am not speaking". "It is possible that I am speaking, but there exists nothing (speaking)" is inconsistent, in contrast.
6. *Symmetry tests* to test whether a claim has content by testing whether it has an applicable negation as well (as [in]famously practised e.g. by Wittgenstein with respect to 'know' one's own mind). This method cannot consist, of course, in demanding that for every concept there *are* entities which do not satisfy it, as otherwise all true generalization will be senseless. A generalization contrasts to another generalization in content in case there are *conceivable* cases (ultimately, consistent models) in which the one is true and the other false. And we may observe as well that negations can be had on the cheap by just negating the whole sentence in which an expression under investigation occurred. The method thus asks whether situations in which the application of the expression was wrong (and thus its negation true) can be conceived, make sense in ordinary usage. In case one type of situation does not allow for applicable negations the method challenges us to

find the proper contrast class of situations (e.g. even in case, let us suppose, we cannot but know our minds, saying “I know I am in pain” makes proper sense if the contrast class are not cases of being wrong about one’s own mind, but cases of knowing the mind of other people or knowing facts). This method, therefore, includes *contrastive analysis* where two concepts are elucidated in complementary fashion. Given such a contrastive analysis one can also ask whether two supposed opposites are really incompatible or not (say determined events or body movements and freely chosen actions).

7. Testing *negating and questioning a sentence* to refute or corroborate its definitional or 'grammatical' character (as practised e.g. by Moore when questioning definitions of “good” or by Malcolm when discussing in *Dreaming* sentences like “I am sleeping” or “I am dreaming”). This method can be added only in case method 6 is understood in the way just outlined. Otherwise all ‘grammatical’ sentences would be ‘meaningless’ as they have no applicable negation. Grammatical sentences remind us of the basic role an expression has, the reason why it is in the language, its definition in case it is a defined expression. They contrast not with their negations, but with the idea of not having *this* expression at all. [Against some aberrations in Logical Atomism and Logical Positivism OLP should not claim that such ‘grammatical’ sentences have no content. Otherwise a lot of analysis ends in sentences without content! Even a humble ‘grammatical’ sentence like “Numbers are not coloured” tells us that under no conceivable circumstances (in no model) numbers are coloured. And OLP should not claim that the negations of ‘grammatical’ sentences, just because they are necessarily false, have no content which can be understood. “The 18th prime number is coloured” and “I found the key the whole morning” have different and understandable – even though contradictory – content: only the second sentence is about a key. Concluding from their inapplicability – apart from a reinterpretation involving conversational implicatures in a specific context of communication – to their meaninglessness would be an instance of confusing use and meaning.]
8. Asking for *truth or assertability conditions* which allow a sentence to have (objective) content and argumentative force in face of an audience (as practised e.g. already by Frege when he casts doubt on sentences about 'the same experience', taken up by Wittgenstein's remarks on psychology). Some statements of truth or assertability conditions aspire to capture the conventions of ordinary usage (i.e. to capture at least in part the proper meaning of a word). Special caution should be given to the treatment of 'exceptions' of use, as classifying counterexamples as 'exceptions' provides easy immunization of a proposed rule of use. There have to be at least partial explanations relating the supposed exception to the core use as derivative. One may attest the oc-

currence of exceptions, as a measure of last resort, given that no better re-constructions of the core rule of use are available.

9. *Rephrasing* to elucidate proper usage and exclude improper philosophical interpretations as added linguistic luggage (as practised e.g. by Ryle with respect to psychology and the ‘systematically misleading expressions’ diagnosed by him, or by Wittgenstein with respect to mathematics). Ultimately the ordinary language philosopher tries to capture in some phrases the core use (and thus the core conceptual knowledge) coming with some expression. Some ways of expressing oneself are clearer and ontologically more parsimonious than others. The resulting sentences should carry with them for all speakers the ultimate degree of linguistic and conceptual certainty. Even if ordinary speakers could not have formulated their implicit knowledge in this form they will recognize these paraphrases as certain, given the philosophical analysis has been successful. OLP makes explicit what has been tacit. By it we gain (explicit) knowledge, but we need not necessarily improve our already given competence. OLP typically resists regulating ordinary language (i.e. issuing new rules of use), but on the other hand one may argue that regulating use is one option present in ordinary language, resting on our ordinary understanding of conventions. In fact, the re-phrasing and re-regulating may aspire to capture with good reasons what ordinary language speakers have regarded already, save the now by the re-regulating excluded improper (philosophical) uses, as proper usage. Hare in *The Language of Morals* calls this ‘suggesting a terminology’ and ‘an analytic model’.
10. *Formal language tools* in OLP can be seen as attempts to explicate perspicuously what lays implicit in ordinary language. By abstracting away from many of the complexities of ordinary language structures can be highlighted (e.g. when in propositional calculus a shallow analysis of propositional structures helps to see propositional consequences). This helps in exploring and understanding the logical structures of language. It should not be confused with the claim that some formal language helps to express something which could not be expressed in ordinary language. This claim reduces itself to absurdity as the formal language has been introduced by informal (i.e. ordinary) talk about it and its workings. We understand the new expressive tools by reading them in the light of the corresponding logical structures in ordinary language. Again, it may be convenient to have formal tools to focus on some structural features, but this does not mean that they could not be expressed less conveniently in ordinary language.

Some of these methods automatically recommend themselves for a *therapeutic perspective* on philosophical claims (as in Wittgenstein's often practised argu-

mentation that some philosophical claim resulted from detaching words or whole sentences from their original context of use and often add the error to assimilate them to other contexts).

One method related to the therapeutic exposition of ordinary usage consists in

11. *Challenging for systematicity* the supposed new supplementary philosophical jargon. Philosophers often revise some words' use and still keep parts of it effecting the supposed continuity to ordinary usage that legitimatises their theories. If they were proposing extensions of ordinary language they should exhibit a systematic usage (e.g. by allowing for contexts warranting or challenging respective claims). Missing such a new systematic usage one can dismiss the deviant use of the ordinary expressions as nonsensical (as practised e.g. by Austin and Wittgenstein when discussing certain forms of scepticism).

Of course not all followers of OLP or conceptual analysis endorse or employ all these methods (e.g. Jackson, as a recent advocate of conceptual analysis, equates it with method 4, hinting at method 9 [Jackson 1998: 42, 46]). The methods assembled present the arsenal of OLP in ideal form.

§12 Defending Paradigm Case Arguments

Of these methods the most controversial has been the appeal to paradigm cases. OLP has sometimes been rejected by first identifying it with paradigm case argumentation and then rejecting this type of argumentation. Although the force of this method, in fact, can be overstated it should not be dismissed. One may delimit its argumentative value in the following way:

If there are criteria to employ an expression, however loosely they are specified or understood (at the beginning), the expression has a justified usage. With respect to such an expression factual and justified usage can be distinguished and come apart. In the extreme case all users might employ an expression contrary to its criteria of employment (i.e. wrongly), because they err on the fulfilment of (some of) the criteria of justified use. In an expansion of the extreme case even the supposed paradigms which introduced us to the use of an expression may turn out to be cases where the expression should not be employed at all! We, and the others who introduced us to the expression, erred on the criteria of justified employment of the expression being fulfilled here. Reference to paradigm cases, therefore, carries profound *prima facie* weight in pointing to justified uses of an expression: absent justified doubt we can trust them – but we cannot treat them as logically not refutable cases. As standards of justified use and justified doubt are contextual the *case for* a scepticism may never arise in practice (especially so for the more extreme versions of scepticism), but the sceptical question at least can make proper sense. It asks us whether any of the supposed items we employ an expression in question to

really fulfils the criteria of its justified use (i.e. whether an expression with that meaning has a non-empty extension). That we have erred in our judgements that an application of an expression is justified does not entail that we have not understood the rules governing the expression. Thus an expression may have a history of an application of the rules governing it, and thus a use in the language community, without ever having been employed correctly.

On the other hand, putting some paradigm cases into doubt referring to the concept of criteria of justified use employs the idea that we in some way understand or process the criteria in question. Once we leave concepts immediately tied to observation, such criteria, to be understood, have to be expressed in sentences of some language. There is no harm in some form of holism which states that this language may be the very same language the expressions of which we are justifying in their application. In any case, we are led to expressions which *themselves* cannot all be employed unjustifiably so far without the idea of confronting a stable usage with failing paradigms and zero extensions breaking down. We can only doubt some paradigms at a time (following this sceptical idea). Pointing to paradigms can at least *shift the burden of proof* (in this case the burden of coming up with justified doubt) to the sceptic. In the light of a *prima facie* correct employment of an understood expression criticism or scepticism has to make clear whether it really concerns the employment of the expression in question – in which case it might be easily refuted – or whether it rather proposes to redraw the limits of the employment of the expression in question (i.e. takes a meta-linguistic stance, as already Malcolm in his controversial exposition of the argument form [Malcolm 1942] observed) – in which case the real issue centres on which language one should use, the new language coming with its own paradigms of usage, of course. Debates on language reform are certainly admissible, but the move to redefining an expression already shows the force of pointing out paradigm cases of the ‘old’ usage of the expression. In some cases the boldly presented criticism or scepticism with respect to ordinary usage may turn out to be a proposal for adding further linguistic resources, and with respect to this proposal the proponent again may have the burden of showing their exigency.

§14 Descriptivism

‘naturalization’ nowadays is often recommended to cure all kinds of philosophical worries and solve old problems. There might be an ‘epistemology naturalized’ and a naturalized philosophy of language. But ‘naturalism’ is understood in quite different fashions. It might be meant as a metaphysical thesis more or the less equivalent to materialism, or it concerns the way of doing things philosophically.

The essential question in the context of discussing OLP is:

(Q1) What is meant by giving a *naturalistic explanation* of some kind of linguistic behaviour?

And assuming some answer to this question, the essential problem is

(Q2) Do these naturalistic explanations *explain* anything at all?

One could think naturalism is more about describing events than explaining them. Descriptions seem to be just the opposite of explanations. One could think, on the other hand, that naturalism in the philosophy of language employs some kind of reductionist explanations of linguistic behaviour (in terms of neurophysiology or whatever is considered as the basic science). But given the anti-reductionist arguments concerning the rule governed nature of using language (especially the *socially mastered rules* of using words to refer to something), one might suspect that these reductionist explanations are at the wrong level of theory building to explain overt linguistic behaviour (e.g., being criticized for using expression α on occasion s) at all. If we ask why we speak the way we speak, an account in terms of neurophysiology, so the argument runs, gives us no reason to understand the patterns of overt linguistic behaviour.

Naturalism is often traced back to the work of (the later) Wittgenstein. I will start with some remarks about Wittgenstein as well. With respect to his analysis in the *Philosophical Investigations* (PI) we should distinguish between ‘strong’ and ‘weak’ descriptivism. Both forms of descriptivism might be seen as answering (Q1). (Q2) is considered in face of the strategic question why ‘to go naturalistic’ in the first place.

According to Wittgenstein philosophy is merely descriptive. There a lots of passage in the *Investigations* stressing this point, e.g.:

“It leaves everything as it is.” (PI §124)²³

“All explanation has to go, and description has to take its place.” (PI §109)

“Philosophy just states things and does neither explain them nor deduce anything from them. – Since everything is laid open, there is nothing to explain.” (PI §126)

Now, this claim of ‘descriptivism’, as I call it, can be understood in two ways:²⁴

(i) *Strong Descriptivism*. Strong Descriptivism claims that philosophy describes *mere* regularities. In its field of investigation (i.e. linguistic communities and their behaviour) there are regularities. Saying that there are rules amounts to, according to (PI §54), watching the events and extracting a law, like a law of nature is extracted from regular behaviour in other fields of sci-

²³ Translated from the German original.

²⁴ I am not going to discuss which of the two variants is closer to Wittgenstein’s opinion. There are a lot remarks congenial to Strong Descriptivism. On the other hand Wittgenstein’s insistence on reasons and the more general problems of an eliminativist view on rule following normativity in linguistic behaviour, which are also not discusses here, favour, on the Principle of Charity, that Wittgenstein himself is closer to Weak Descriptivism. See also his remarks on frameworks in *On Certainty* which point towards Weak Descriptivism.

entific investigation. The very term “law of nature” is used here by Wittgenstein. Natural laws are, of course, regularities. The objects for which the law of nature holds behave as the law tells us, but these objects do not *orient* their behaviour on the law. They do not *consult* the law to confirm to it. Natural laws are not rules for the objects under these laws. They do not have to be consulted to keep in force. And the observer of such laws need not himself understand the laws or make them the laws of his behaviour. So philosophy lays open the facts that speakers naturally behave in this or that fashion. The opinion that language is a rule governed normative behaviour overlooks, according to Strong Descriptivism, that meaning and reference are fixed by our natural traits. The normative idiom (of rule following) is therefore dispensable. Philosophy *cannot* do more than clearly describe regularities of linguistic behaviour. Who does not recognise this is caught in mistaken pictures and needs therapy. (The business of reduction or giving a systematic theory of the laws involved need not be part of philosophy.)

(ii) *Weak Descriptivism.* Weak Descriptivism originates as a restriction of the claims of Strong Descriptivism. The main weakness of Strong Descriptivism is the impossibility, which is not argued for here, but which is widely recognised, to forsake all rules of argument and speaking. The attempt to forsake all rules and normative claims (in using the intentional idiom) seems to be both self-refuting and against some of our most embedded intuitions. Weak Descriptivism tries to combine the strength of the naturalistic, descriptivist approach with the thesis that linguistic behaviour is rule following behaviour (i.e. that speakers orient themselves on rules or conduct their linguistic acts in a way to comply to these rules). The strong point of descriptivism is that philosophy leaves everything as it is. Nothing has to be constructed to justify some philosophical claim. The basic structures of our intelligent behaviour are just read off from an exact description of our linguistic behaviour. And these structures are justified by the fact that the practise which exhibits them is successful. Alternatives (including alternative philosophical claims on intellectual standards) stand on a far less firm ground by not being entrenched in our successful way of life (‘life form’ as Wittgenstein might say).

And at the same time these descriptions can speak of normativity – for the simple reasons that normativity is present in the observed behaviour: If someone is to describe the linguistic behaviour of a community, she has to describe the rules/norms which govern and constitute this very behaviour. By being described norms do not cease *to be* norms! A statement referring to a norm (a statement about a norm) is true only if the norm *is* in force in just that way the statement is saying it is.

Wittgenstein, for example, once and again stresses the fact that a linguistic community evaluates some behaviour as “correct” and others as “wrong”. These evaluations would make no sense if the person whose behaviour was evaluated as “wrong” could not reorient her behaviour on the communal stan-

dard. For the observer of this community this means, as Peter Winch has elaborated (Winch 1958), that she understands *why* somebody is criticising somebody else. The observer at last can participate in the observed behaviour. Understanding rules of usage has to be more than a merely inductive generalization of observed speech behaviour, however vague and not spelled out the shared common knowledge might be. All this means that speakers orient themselves on linguistic rules which are more than mere regularities. A cat might develop a regularity responding to similar circumstances (e.g. the alarm clock went off) with similar behaviour (e.g. mowing for breakfast), but there is no intersubjective standard to which the cat's behaviour confirms. Each new twist modifies the regularity. The description just records this factual regularity and its development over time. An intersubjective rule, in contrast, can be observed to be kept in force by evaluations of correct and incorrect behaviours. At least one's implicit knowledge of shared conventions has to be at play in the 1st person case: I cannot model my own competence by inductive generalizations of my past linguistic behaviour. Apart from the difficulties of such a monological enterprise close to the issues of private language use, it does not capture the difficult to capture process of coming to participate in shared linguistic knowledge.

So descriptivism in the form of Weak Descriptivism does not exclude viewing linguistic behaviour as normative. The criticism one might level against naturalism on this point does not apply here.

So how does Weak Descriptivism answer (Q1)?

If linguistic behaviour is rule governed, a systematic description of it is adequate only if the observer has understood (and included in her description) what the standards are and how the standards are enforced. And having understood the rules governing the linguistic behaviour the individual behaviour is straight forwardly *explained using these rules* as (part of the) premises.

The behaviour is explained on the level of linguistic 'laws'. An anti-reductionist should have nothing to complain here. Only a reductionist might complain that this is not *enough* explanation. Seen this way, Weak Descriptivism, although being a form of naturalism, is anti-reductionist!

Why should we take the attitude of Weak Descriptivism? I will consider one example: the 'division problem' as thought of by Eli Hirsch (1993). Hirsch is concerned with the idea of (natural) kinds. He introduces the thought experiment of different kinds of 'strange languages'. Strange languages divide reality in kinds and individuals in ways completely different from our normal languages. Strange languages seem to be bizarre, seen from the point of view of our language. They might introduce kinds disjunctively (i.e. 'introduce' from the point of view of our language, in the strange language these kinds are, of course, not disjunctive, but just given). So a strange language might contain the kinds *cathouse* and *housecar*. Seen from our language they can be defined:

Cathouse(x) := x is a cat or x is a house

Housecar(x) := x is a house or x is a car

This language has the same expressive power as our own, since our ordinary kinds can be defined within this language:

Cat(x) := Cathouse(x) \wedge \neg Housecar(x)

House(x) := Cathouse(x) \wedge Housecar(x)

Car(x) := Housecar(x) \wedge \neg Cathouse(x)

Now, this strange language has *less* kinds or kind terms than our language. So this language seems to be simpler than our language. It carries less ontological commitment! For the sake of ontological simplicity we *should speak* this language, but this sounds absurd.

This is (part of) the division problem. How can it be explained that we do not speak a strange language? A non-naturalistic solution could be a metaphysical theory of *natural* kinds which could disqualify *cathouse* and *housecar*. This requires an ontological account of naturalness which might be no easy exercise (cf. Hirsch 1993: 53-78). And with respect to this ontological theory there still needs to be explained why our language structure would follow naturalness, if there is such a thing in reality.

A solution could be found turning to naturalism (in the form of Weak Descriptivism): The strange language is to be rejected since we *are* built as we are built (i.e., our language faculty is structured in some definite way), and we have the habits we have. The structures of our language faculty (especially our habits of categorizing) do not allow strange languages. We have to consider them strange. So Weak Descriptivism would describe the standards of our categorization behaviour: evaluations what speakers consider strange explain why there are *cat*, *house* etc. around, and not *cathouse*, *housecar*.

Weak Descriptivism can explain what we do according to the standards it described. It leaves the rationality of our behaviour intact. Explanation occurs *within* the framework taken for granted. The rationality of it is there – in Wittgenstein's words in *On Certainty* – “It is there – like our life”.

But is this really an explanation? Hirsch (1993: 116) complains that the naturalist would just give a vacant thesis that we were just that way and would give no further argument for this to be the case. One might ask “Okay, but *why* are we built this way?”

This sounds a bit like (Q2). Nevertheless, this accusation of naturalism misses the whole point of ‘going naturalistic’. Naturalism is pursued since *a priori* arguments to solve some problem have failed. Their failure is the basic reason that only a naturalistic account – instead of a sceptic agnosticism – can answer to the problem. To go naturalistic *means* that one is referring to facts or describing facts which cannot be questioned further. Strong Descriptivism might try to give an explanation of our behaviour by referring to facts outside of the

way of life described. But these explanations no longer answer to the questions put within this way of speaking and acting. Seen from this perspective of participants in this way of speaking we might leave behind this kind of naturalistic investigation altogether.

A systematic description in the sense of Weak Descriptivism can amount to a 'rational reconstruction' of our intuitions in the field in question. Question (Q2) itself is not as obviously relevant as it seems. It could rest on seeing the fact that we can ask for explanations of the framework itself from without as an insufficiency of giving reasons from within: the "why" in question (Q2) really is no further "why" of the sort answered by Weak Descriptivism, but a different "why" altogether. Knowing why we are biologically or neuro-physiologically build the way that we are build might be of no great relevance to the questions raised.

This might be the idea of PI §655: "It isn't a question of explaining a language-game by means of our experiences, but of noting a language-game."

Reductionist naturalism might be more successful with respect to this further investigation in the natural history or the causal antecedents of our (linguistic) behaviour than a mere description. In a wider scientific perspective on linguistic behaviour we probably might be interested in reductionist explanations. Nevertheless Weak Descriptivism seems to be a first option.

That there might be different attitudes and aims in naturalism has been noted before. The distinction made here between Strong and Weak Descriptivism is related, for example, to Strawson's distinction between 'strong naturalism' and 'liberal or catholic naturalism' (Strawson 1985). Nevertheless many arguments don't seem to see the distinction.

This is unfortunate because the merits of Weak Descriptivism aren't appreciated by conflating it with Strong Descriptivism. The worth of the distinction, therefore, could lie in considering what kind of naturalism might be required or asked for in case opting for a naturalistic solution seems to be the most promising option at hand.

5.3 Epistemology and Ordinary Language Philosophy

As an illustration of some of the afore mentioned methods of OLP this section deals with a couple of epistemological issues: the definition/analysis of the concept of knowledge, principles of knowledge attribution, where to place contextualism in epistemology (on knowledge or justification), the resulting stance on scepticism, and some special questions surrounding the epistemology of religious language.

§15 Knowledge as True Belief

“a knows that p” entails p, because we cannot know what is not the case. “a knows that p, but p is not the case” is paradoxical. Knowledge is veridical. Formally: $Kp \supset p$.

“a knows that p” also entails “a believes p”, although for reasons of maximal information content one can say, criticising a non-maximally informative statement, “a does not believe p, a knows p”. Knowledge involves subjective conviction, at least of the strength of rather believing p than $\neg p$. For this reason a lucky guess does not count as knowledge. Just by guessing I may hit on a truth, but I will not form even a weak subjective *conviction* that p. This excludes examples of lucky guesses as counterexamples to defining knowledge as true belief. In these cases we just do not believe that p, but *only* guess. We can see the difference, for instance, in somebody’s behaviour: believing p includes being prepared to act on the assumption p, which in general (i.e. except in cases of taking severe risks) is not true for guessing that p.

We can thus define, using the obvious abbreviations:

$$(K) \quad Kp \supset Bp \wedge p$$

One may also be *convinced* that p in the sense of not deeming it (epistemically) possible (in the context) that p. To distinguish this stronger case of belief from just rather believing p than $\neg p$, one may prefer defining knowledge as true conviction. Using “Cp” for “being convinced that p”:

$$(K_+) \quad Kp \supset Cp \wedge p.$$

Beliefs can be strong beliefs or weak beliefs. The common use of “belief” covers both types.²⁵

Both have to be distinguished from deeming something (epistemically) possible, which allows for deeming it possible that p and deeming it possible that

25 Following common usage I continue to talk generally of knowledge as 'true belief', as long as disambiguation to knowledge as 'true conviction' is not needed. I use “convinced” and “conviction” only in the way just introduced, but use “being certain” in a way that admits of degrees of subjective certainty (i.e. so that it may include cases of weak belief).

$\neg p$. As being certain to a stronger degree implies being certain to a lesser degree, conviction implies belief:

$$(D) \quad C_p \supset B_p$$

Thus (K) expresses the more general definition of “knowledge”.

§16 Is Knowledge More Than True Belief?

Often knowledge is defined as 'justified true belief', and a whole industry has developed dealing with the famous 'Gettier-cases' (Gettier 1965), which show examples of people with justified true beliefs whom many epistemologists would not classify as people who 'know' as there is no (causal) connection between their justified beliefs and the truth.

One option to deal with this is to expand the definition of knowledge above to 'non-accidental true belief'. Cases of reliable, though non-conscious, belief formation invite this weakening of 'justified true belief'. If this was the default understanding of knowledge the counterexamples could be excluded as they all involve accidentally true beliefs. One may also use “warrant” for the inclusive category and “reason” for the intentional doxastic subcategory. “Knowledge” could then be defined as “warranted true belief”.

One may question, however, the 'Gettier-cases' in the first place. “Paul knows accidentally that the cat was in the flat” had to be paradoxical if knowledge required warrant or non-accidental connections between the convictions and the facts. As well we can non-paradoxically say: “Paul knew that the cat was in the flat, but had no justification for it”. We can even say, without obvious self-contradiction: “Paul knew that the cat was in the flat, but all the reasons he gave for it were wrong”.

As the requirement of having a belief – not to mention a conviction – excludes cases of lucky guesses there is no problem with counting the 'Gettier-cases' as cases of knowledge. In the presence of subjective certainty (at least to the degree of rather believing p than $\neg p$) even unjustified true beliefs or true convictions count as knowledge: “He knew it, but could not tell why and how” is a proper report.

So we should follow ordinary usage and reject both defining “knowledge” as “justified true belief” as well as “non-fluky true belief”.

As a result the notion of knowledge may lose a lot of interest to the epistemologist. That may well be so. Justification and warranted belief are epistemologically far more interesting than knowledge (attribution). The whole debate following the 'Gettier-cases' put the emphasis doubly on the wrong spot. We are interested in proper justification and other reliable methods of belief formation as they are conducive to those beliefs being true (i.e. being knowledge). Justification in contrast to other reliable, but maybe sub-doxastic, ways of belief formation also helps us to understand the world, to see structural dependencies. If knowledge is the sole aim of our epistemic rationality, justifica-

tion will be a means to warrant our claim to have achieved this aim, and as a *means* will be not part of the definition of knowledge (cf. Sartwell 1992). If understanding the world and ourselves within it is another aim of our epistemic rationality, then justification may be an integral part of this aim, but need not be an integral part of our other epistemic aim, namely having knowledge. It may be difficult to spell out what ‘understanding’ is, but one might argue that whereas knowledge is the original and fundamental epistemic aim as it is valuable for survival, understanding is an advancement in reflective epistemic rationality, expressed in such ideals as the ‘*bios theoretikos*’. Philosophers as preoccupied with justified theories and beliefs may just be occupational myopic in identifying the two.

§16 Attributing Knowledge

Attributing knowledge involves a stand on the facts by the attributer. We can only attribute knowledge that *p* to George if we believe that *p* ourselves. Usually we assume a God's eye-view in these cases, as this allows us to judge the quality of George's warrants for his beliefs, in case he has any, objectively.

Uttering “I know that ...” can be used emphatically to express subjective conviction that something is the case, employing that knowledge entails truth. Answering somebody with “I know that” signals agreement and frees the other of providing reasons to convince me. Third person reports “He knows that ...” also express agreement on the subject matter.

Somebody who truly reports of somebody else that she knows that *p*, knows *p* herself (as expressed in Hintikka’s ‘transmissibility of knowledge rule’ [Hintikka 1962]).

Whether they express more depends on the analysis of knowledge (i.e. whether the attribution also involves the attribution of having access to justifying good reasons or not accidentally truly believing the subject matter). Somebody who reports of somebody else that she knows *p*, should as a matter of consistency of her own beliefs supposedly believe that she herself knows that *p*, too.

First person plural reports “We know that ...” express or appeal to common knowledge.

In the presence of positive introspection (believing, at least dispositionally, that one believes *p* in case one believes *p*) and definition (K) one *cannot* believe or even utter “I believe *p*, but I don’t know *p*”, since as one believes *p* the only reason for not knowing *p* can be $\neg p$, and knowing $\neg p$ or having sufficient evidence that $\neg p$ excludes believing *p*. The sentence “I believe *p*, but I don’t know *p*” is not logically contradictory, but conflicts with the felicity conditions of rational belief and assertion. The consistency of it can be shown by modalization [method 5]: “It is possible that I believe *p*, but I don’t know *p*” is not inconsistent, but makes perfect sense, and is often true. The same can

be said about “Georgie is not sure that p , but she knows that p ”. Georgie cannot successfully assert “I am not sure that p , but I know p ”, as claiming to know p involves being convinced that p [see below].

Positive introspection with respect to belief may be controversial. Suppose we accept it. Then positive introspection with respect to knowledge has to be accepted as well in its externalist validity. “ $K \supset KKp$ ” has to be accepted given positive introspection for belief, since in case one believes p and p is the case (whether recognized by oneself or not), one knows that p , and, given positive introspection for belief, one believes that one believes p (i.e. knows that one believes that p). This does not sum up to believing *oneself* that one knows that p , as p being the case may be beyond one’s ken, but as it *is true* that one knows that p , Kp and KBp combine to KKp . Whether one stipulates “ $Kp \supset KKp$ ” as an axiom for a theory or logic of knowledge depends then on whether the sentences of the forms “ Bp ”, “ Cp ”, “ Kp ” are to be taken as being 3rd person reports only or not. Taken as a logic of epistemic states or reports of epistemic states, in contrast to a logic of autobiographic expression, epistemic logic may contain the positive introspection principles.

The iteration principles should be coupled with a dispositionalist reading of the epistemic or doxastic reports: one who dispositionally believes p may well have the further disposition to believe that he believes p , whereas it seems psychologically and introspectively questionable that somebody who has an occurring belief that p has all accompanying higher order occurring beliefs as well. Of course the epistemic and doxastic operators have to be read unambiguously in the iteration principles. Once we understand “Peter knows p ” as allowing for Peter not recognizing his knowledge himself, then even autobiographic positive introspection can be endorsed. In case I recognize my knowledge I also confirm positive introspection, because in this case I know that I know. If I know without recognizing my knowledge, by positive introspection I know that I know p , but although this includes that I have the disposition to believe that I know p , it does not require that I recognize this belief (and thus my knowledge that p).

That knowledge is veritative (i.e. satisfies “ $Kp \supset p$ ”) does not mean that it is transparent to oneself in a non-dispositional sense (e.g. by being testified by occurring meta-beliefs). “Maybe I know” is a perfectly acceptable statement, like “I can’t tell whether I know this”. Maybe I cannot tell whether I know the shortest route, but later I recognize that I knew it. My non hesitating manner of driving showed that I believed to know the shortest route. So *in fact* I knew that I knew.

All attributions of knowledge are true only if the supposedly known state of affairs is a fact.

§17 Knowledge and Justification in Contexts

Knowledge *if* tied to warrant depends in its justificational requirements on the context. The standards of having warrant or giving justifications vary widely between contexts. We only have to have warrant sufficient in terms of the contextual standards, for example claiming a surface to be “flat” in managing football pits may be quite different from doing this in manufacturing. Accordingly the warrants have only to defeat doubt in terms of the contextual standards. Some doubts are just too far-fetched (e.g. using a microscope in managing football pits). Many varieties of of 'scepticism' violate this contextualism. They try to impose standards foreign to a context, and thus go against ordinary usage and established epistemic practices.

We give reasons without going back to some ultimate ‘given’ or ‘incorrigible’ facts, we refer to other reasons instead, even if this means going in circles, given that the circles are wide enough. Reasons (statements held to be true) support other reasons/statements, but we *can* always put the last reason given to test.

Nevertheless, we can solve this regress as follows: Because of the holistic procedure of justification (and therefore of meaning something) we are allowed to keep asking for further reasons *in principle*, but in doing so we employ a meta-rule of *sufficient foundations*:

(SR) If there is no founded/reasoned doubt, there is no need for further foundation/argumentation.

We employ semantic rules and justificatory procedures in some situation and try to conform to the habits of our community. If someone asks us why we assert a specific sentence in some situation, we justify our claim by reference to the fulfilment of the criteria of use (i.e. give reasons by citing a second description like “feline mammal” when justifying the use of “cat”). This duty is part of the commitments of assertion. But if in respect to the fulfilment of some criteria in such an argument after several steps there is no longer reasoned doubt (i.e. no foundation for belief in their non-fulfilment), why should we proceed in founding our claims? Our argument now is (relative to all claims founded in *that* debate) sufficient. Relative to our knowledge of this state of the argument and our knowledge of the meaning of the expressions involved it is the optimal logical procedure to evaluate the usage of expressions as ‘correct’, and our claim as ‘justified’. All reasons we have now speak in favour of this evaluation. This is neither an act of decisionism nor just an act of some opaque capacity, but the application of our rule-following procedures which can be taken up again in principle and has been interrupted only at a sufficiently clear point. So assuming the meta-rule (SR) seems to be the lesser evil in comparison to the consequences of some type of fundamentalist epistemological stance, which invites scepticism. We have *not overcome* the principled problem of the regress, but we can see that it is *harmless* if we employ our rule of sufficient foundation. The regress problem has our (not yet re-

flected) intuition of foundation as its driving force. But with respect to our reflected intuitions the meta-rule as a principle that an argument just has to be *sufficiently* clear seems equally strong.

Some proponents of knowledge as justified true belief have concluded that knowledge is contextual. This places contextual dependence in the wrong spot. Knowledge as true belief is an all-or-nothing matter: one knows or one does not. Justification is context dependent, and the principled reason for this rests in the holistic nature of justification: as we could in principle always proceed with further justification we need contextual standards of sufficient justification. Analysis reveals the proper distinction here between knowledge and justification.

§18 Assertion and Justification

Duties to justify one's statements stem from the commitments of assertion (i.e. from the felicity conditions of the speech act of assertion). They affect all one's assertions, whether or not they carry the additional modifier "I know that". Even asserting *p* without any ado carries the commitment to be able to give some (minimal) justification. Thus this commitment does not come with knowledge claims alone or is tied only to self-ascription of knowledge.

Keeping apart semantic and pragmatic features leads one to see in assertions on the one hand (eternal) sentences which are claimed to be true, where truth can be understood in terms of correspondence, and on the other hand the assertive force (i.e. making an assertion stating a sentence to be true) tied by the felicity conditions of the speech act of assertion to the idea of justification, where the primary understanding of justification links being justified with being in a disposition to be able to at least partially provide reasons for the claim in question. A semantic correspondence theory of truth, as the hallmark of all versions of realism, combines thus naturally with a coherence or pragmatist theory of justification, as the hallmark of several versions of internalism.

Justification comes in several varieties (*inter alia* being justified by having doxastic access to reasons, as well as being justified by being cognitively equipped with reliable procedures of hooking up with the world).

The combination reflects both the liabilities of a speaker conforming with the conventions of an honest assertion (that assertions cannot be put unjustifiably forth *ad libitum* without risking one's credibility and trustworthiness) as well as the conception of objective truth at least conceivably outstripping our resources of justification.

Whereas asserting *p* entails "I am (sufficiently) certain that *p*" – obviously the converse entailment does not hold – asserting *p* does *not* entail "I know that *p*", as we know by conceptual reasons that conviction does not entail knowledge. Self-ascriptions of knowledge involve further warrant (e.g. quoting tes-

timony by others or reference to shared data) or back-references to somebody else's claim (e.g. in a consenting "I know").

Look at the following claims:

- (1) I assert that p
- (2) I assert that p is true.
- (3) I assert that I know that p .

Claims (1) and (2) are logically equivalent given both a correspondence theory of truth (as the complements are logically equivalent in it) as well as in a justificationist theory (as warranted assertion involves justification enough to treat the complements as materially equivalent).

(1) need not entail that I am convinced that p . So an easy argument to (3) is blocked.

The argument is this:

Suppose that by (1) – and (2) – I am convinced that p , and I have some warrant for it. And by minimal introspection I believe that I am convinced that p . By (2) I am convinced that p is true. So given my knowledge of the definition of "knowledge" I should endorse (3), since some limited form of conjunctions of convictions and at least closure of convictions under definitional substitutions may be viable principles of reasoning. In practice at least we more often than not act as if we knew that p , in case we are convinced that p . In some cases we prepare for the eventuality of error. It seems that the conceptual distinction between belief or conviction and knowledge hinders us in taking (1), (2) and (3) as logically equivalent, but given conjunction of strong conviction and positive introspection we can reason:

- (4) Cp PREM
- (5) CCp Positive Introspection, 4
- (6) $C(p \wedge Cp)$ Conjunction, 4, 5
- (7) $CK_{+}p$ Definition, 5

Conjunction may fail for a weaker notion of belief (as, for example, degrees of probability, although individually above the threshold of 0.5, conjunctively may fall below this threshold). But for those claims I am convinced of, for which, insofar as conviction does not come without warrant, I am also in a position to assert them, we can reason to (7) and also have the additional option to assert that we are convinced. (1) and (3) should be treated interchangeably then. Assertion (as operator or illocutionary force) should support the following argument:

- (8) Ap PREM
- (9) ACp PREM
- (10) $A(p \wedge Cp)$ Conjunction for assertions, 8, 9

(11) AK_+p Definition, 10

End of Argument.

Because the argument moves from being certain to being convinced, and relies essentially on (K_+) and related principles for “C” it cannot establish the general entailment of (3) by (1) for (K) . But it tells us something about our habitual ways to express subjective certainty, namely proceeding from (1) to (3). This might be taken as the test of being convinced: you are only convinced if you assent to: “Do you know that...?”

One should again not overlook that knowledge ascriptions and standards are contextual, and this contextuality enters here (i.e. also in [7]) in form of some standard of justifying the conviction in (4). Given a context I equate my convictions with *claims* of knowledge that I am ready to assert. Conceptually I do not, since even conviction does not *entail* truth, equate claims of knowledge with knowledge. (7) and (11) are not logically equivalent with

(12) I know that I know that p.

– apart from using (12) again *emphatically*.

Given the argument above (3) can serve as an expression or indication of conviction in contrast to an expression or indication of mere weak belief.

Given that we do not use (1) and (3) interchangeably (K_+) cannot be the sole or generally employed definition of “knowledge”.

§19 Negative Introspection

In the preceding part of the chapter knowledge was understood as true belief or as true conviction, and nothing else. This understanding of knowledge plays a role in standard epistemic logics. As we said [in Part I] that ordinary language philosophy does not by itself exclude the methods of formal analysis this paragraph uses the means of formal (epistemic) logic [method 10] to argue that understanding knowledge as true conviction is incompatible with the attribution of a faculty of *negative introspection*.

Autoepistemic reasoning is reasoning the inferences of which depend on representing one’s own state of belief. A cognitive agent engaged in autoepistemic reasoning draws conclusion from introspective beliefs. Such epistemic beliefs express that the cognitive agent has this and that non-epistemic beliefs. If agent *a* has the belief “The cat is on the mat” the introspective belief is “I believe that the cat is on the mat” or – without self-representation – “It is believed that the cat is on the mat”. Formally this can be expressed using epistemic modal operators like “B” (for belief) or “K” (for knowledge) above.

One question may be now, how much access and how reliable access some cognitive agent *a* has to its non-epistemic beliefs (typically called ‘first order beliefs’ as they do not involve epistemic operators). Let *B* be the set of the

agent's beliefs. An agent with ideal self-access or ideal introspective capacities may fulfil both of

- (i) *positive introspection*: $\alpha \in B \Rightarrow B\alpha \in B$
- (ii) *negative introspection*: $\alpha \notin B \Rightarrow \neg B\alpha \in B$

Further on, the ideal agent may also fulfil some version of logical omniscience or deductive closure with respect to its first order and even autoepistemic beliefs (as a sentence α may contain epistemic operators):

- (iii) $\vdash \alpha \Rightarrow B\alpha \in B$
- (iv) $\vdash (\alpha \supset \gamma), B\alpha \in B \Rightarrow B\gamma \in B$

For human agents this seems way to unrealistic: neither do we believe or know all logical truths, nor are our beliefs closed under logical consequence. The principles of positive and negative introspection can also be expressed as principles of iterating epistemic modal operators:

- (v) $B\alpha \supset BB\alpha$
- (vi) $\neg B\alpha \supset B\neg B\alpha$

One can now recognize that they are epistemic variants of the modal axioms²⁶ characterising the alethic modal systems S4 and S5:

- (vii) $\Box\alpha \supset \Box\Box\alpha$
- (viii) $\Diamond\alpha \supset \Box\Diamond\alpha$

These are the stronger modal systems. Especially negative introspection seems to require that we believe of *all sentences of the language* that we have no corresponding belief iff we do not have such a belief.

For technical systems (artificial cognitive agents) this might be feasible. If we consider a database, we may say that the facts stored in the database are its first order beliefs. A query is a form of introspective access. If the queried fact is stored the positive reply exhibits positive introspection, a negative reply exhibits negative introspection.

Some try to defend the introspective principles by distinguishing between occurring, dispositional belief and maybe implicit belief. They claim then that we have at least the implicit belief that we believe if we have an occurring belief. All operators, however, have to be read the same way. And then we find a kind of dilemma: Positive introspection seems to correspond to the conscious awareness of an occurring belief. As positive introspection has to apply to any belief, however, we face then an infinity of ever more involved meta-beliefs (just insert ' $B\alpha$ ' for ' α ' any time you like in (v) or (i)). We do not find this infinite hierarchy of meta-beliefs *occurring* in us. We may accept an infinite *disposition* to answer any questions about our (meta-)beliefs with an iterated be-

²⁶ Substitute $\neg\alpha$ for α in (vi) and remember that $\Diamond\alpha \equiv \neg\Box\neg\alpha$.

belief-operator. For dispositional belief positive introspection seems less damaging, but it also seems less compelling. Why should we always have a reliable disposition to a (further) meta-belief corresponding to just any dispositional belief? Our introspective access to occurring beliefs has nothing to do with this capacity. The best argument for positive introspection appeals to our capacity to revise and update our beliefs constantly. Update and revision aim at a coherent set of beliefs and require some form of doxastic access to our beliefs. A database fulfils positive introspection because of a reliable look-up in limited fact storage. Humans may not be built that way, but in general coherent acquisition of new beliefs supports that humans possess some form of positive introspection. Above principles of positive introspection have been defended as part of a 3rd person epistemic logic. They *report*, in case of “ $Kp \supset KKp$ ” essentially from an externalist perspective, of autoepistemic reasoning. Nonetheless, as I know of their validity, I may ask what they mean for my reasoning and my epistemic states.

Negative introspection looks much worse than positive introspection, especially when combined with deductive closure: By recognising that you do not believe γ , but believe α , you will immediately know that γ does not follow from α (given your other beliefs as well)! As we also have false beliefs this does not amount to a decision procedure, but if some cognitive agent had *no* contingent beliefs at all, but fulfilled both (closure and) the introspection principles, that agent would constitute some kind of a decision procedure for *any* underlying logic, which should give as a pause.

The procedure would be the following: The sentences of a language L are recursively enumerable (by some Turing machine M_1), for good measure the theorems of some undecidable logic Δ expressed in L are recursively enumerable (by some Turing machine M_2). Let M_1 provide a sentence α . Check: $B\alpha \in B$? By the introspection principles, especially negative introspection, the agent comes up with an answer. If the answer is ‘No’ this can only be, because α is not believed [contraposition on positive introspection], thus ‘ $\neg B\alpha$ ’ [by negative introspection], and we know that α is not a theorem, thus the non-theorems are enumerable. Combining this with positive introspection or the workings of M_2 provides us for any sentence α with an answer whether in $\Delta \vdash \alpha$ or $\not\vdash \alpha$. This does not provide a decision procedure in the strict sense (and thus no refutation of or contradiction to the undecidability theorems) as the checking procedure certainly is not *algorithmic* – put otherwise: it *cannot* be algorithmic on pairs of contradicting undecidability theorems. Real databases despite their supposed introspective capabilities provide no such problem as they are finite and undecidability comes only with infinite domains (here: infinite belief storage spaces).

Introspection principles are *controversial* in light of human epistemology and human self-access. They are *disastrous* if combined with a strong concept of

knowledge. The strong concept of knowledge defines “knowledge” as true conviction:

$$(K_+) \quad K_+\alpha \equiv C\alpha \wedge \alpha$$

Many see (K_+) as the natural starting place for epistemic logic (e.g. Lenzen 1980). [This agrees partially with our discussion in §§15-18. Remember: ‘conviction’ is understood here as strong belief (i.e. one is convinced that α if $\neg\alpha$ does not seem epistemically possible to one).]

The introspection principles are then assumed for both belief and conviction.

What other principles may now be demanded for the “K”-operator?

The definition and our ordinary understanding give the (T) axiom:

$$(T) \quad K_+\alpha \supset \alpha$$

Following the heuristic of looking at alethic modal logics and a prior discussion of the logic of conviction, the (K) axiom and the idealization of logical omniscience (and closure) may be added:

$$(K) \quad K_+(\alpha \supset \gamma) \supset (K_+\alpha \supset K_+\gamma)$$

$$(RK_+) \quad \vdash \alpha \Rightarrow \vdash K_+\alpha$$

For being convinced Lenzen like other epistemic logicians adopts both introspection principles:

$$(C3) \quad C\alpha \supset CC\alpha$$

$$(C4) \quad \neg C\alpha \supset C\neg C\alpha$$

Given (C3) and the definition (K_+) we get²⁷ the (S4) axiom, positive introspection, for strong knowledge:

$$(S4) \quad K_+\alpha \supset K_+K_+\alpha$$

What about the (S5) axiom, negative introspection? The formula would be

$$(S5^*) \quad \neg K_+\alpha \supset K_+\neg K_+\alpha$$

Translating this using the definition (K_+) we get:

$$(S5^{*'}) \quad \neg(C\alpha \wedge \alpha) \supset \neg(C\alpha \wedge \alpha) \wedge C(\neg(C\alpha \wedge \alpha))$$

This is unacceptable. The usual way in which we fail to know is being convinced that α , but α not being the case: $C\alpha \wedge \neg\alpha$. Assume this to be the case. Then the antecedent is true thus we have the consequent. Again the first conjunct of the consequent is simply true. But we have the second conjunct as well. By propositional logic “ $\neg(C\alpha \wedge \alpha)$ ” is equivalent to “ $\alpha \supset \neg C\alpha$ ”. Now by the (K) axiom for “C” we have:

$$(2) \quad C(\alpha \supset \neg C\alpha) \supset (C\alpha \supset C\neg C\alpha)$$

²⁷ “ $K_+\alpha$ ” is “ $C\alpha \wedge \alpha$ ” this implies, by (C3), “ $CC\alpha$ ” and using this and the (K) axiom for “C” we get: $C(C\alpha \wedge \alpha) \wedge (C\alpha \wedge \alpha)$, i.e. $K_+K_+\alpha$.

Thus in our assumed case, in which we have $C\alpha$, we can detach to get:

$$(3) \quad C\alpha \wedge C\neg C\alpha$$

In combination with (C3) we finally arrive at a contradictory conviction:

$$(4) \quad CC\alpha \wedge C\neg C\alpha \quad \text{or} \quad C(C\alpha \wedge \neg C\alpha)$$

Thus (S5*) has to be rejected. Only because we have a wrong conviction we certainly have not a contradictory conviction, $C\perp$.

Lenzen (1980: 62-65; cf. also Lenzen 1979) now is not content to have **S4** as the logic of K_+ . He introduces a weakened version of negative introspection, which corresponds to the characteristic axiom of the modal logic **S4.4**.

$$(S4.4) \quad \alpha \supset (\neg K_+ \neg K_+ \alpha \supset K_+ \alpha)$$

Or equivalently:

$$(S4.4') \quad \alpha \supset (\neg K_+ \alpha \supset K_+ \neg K_+ \alpha)$$

This second version shows that we have a weakened form of negative introspection here, one concerning only obtaining facts α . If we read α as expressing that α is a fact (more precisely, that the referent of α is an obtaining state of affairs), and read $\neg\alpha$ as expressing that α is not a fact (more precisely, that α does not refer to an obtaining state of affairs), or the negative fact that α 's content is not given, then we can re-formulate (S5*) as negative introspection for both types of facts:

$$(S5^{**}) \quad \alpha \vee \neg\alpha \supset (\neg K_+ \alpha \supset K_+ \neg K_+ \alpha)$$

We have rejected this principle that does not distinguish obtaining from not obtaining states of affairs, or true or false α , why should we accept (S4.4') then?

In fact, (S4.4') is as unacceptable as (S5**). To see this assume α to be true because it refers to some obtaining fact for which you are convinced of the opposite (say the fact of the amount of the grains of sand on Mars being even, while you by accident or your queer astronomical methods are convinced that that amount is odd). Now by α being true we can detach the consequent of (S4.4'). As your conviction is contrary to the facts you do not know α , i.e. $\neg K_+ \alpha$. Thus we can detach again and get: $K_+ \neg K_+ \alpha$. You simply have the knowledge that you do not know α . As you have the conviction $C\neg\alpha$, by positive introspection you know that you have that conviction: $CC\neg\alpha$. If you ask yourself know "Why do I not know α ?" the obvious answer available to you by introspection is "Because I have a contrary conviction". Having understood this much you simply negate your conviction and come to be convinced that α , $C\alpha$, which again means, α being the case, that now you come to strongly know α ! By logic and the obtaining of facts alone one thus comes to revise any old wrong conviction that one has! How could anybody ever have a wrong

conviction then anyway – at least have one for an extended period? Therefore, (S4.4) must be rejected. The logic even of K_+ has to be weaker than **S4.4**.

The argument might be partially defused by not admitting the autoepistemic access mirrored in the question “Why do I not know α ?”, i.e. by stressing the 3rd person character of the iteration principles. But even then we are left with a cognitive system that could subdoxastically (by implementing the introspection principles and coherent updating of beliefs and convictions) reason to the conclusion. The theory or logic of epistemic states will still be empirically unacceptable.

And if that is not bad enough, more trouble is to come. $K_+\alpha$ obviously implies $\alpha \wedge \neg K_+\neg K_+\alpha$.²⁸ Combining this with (S4.4) gives us:

$$(5) \quad K_+\alpha \equiv \alpha \wedge \neg K_+\neg K_+\alpha$$

If we now write the definition (K_+) beneath this

$$(K_+) \quad K_+\alpha \equiv \alpha \wedge C\alpha$$

we immediately see that in the logic of strong knowledge we have the theorem:

$$(6) \quad C\alpha \equiv \neg K_+\neg K_+\alpha$$

And this, again, is just bizarre: Assume α to refer to some state of affairs you have never thought of, maybe because it is way beyond your human knowledge. As you have never thought about α you certainly do not know α , i.e. $\neg K_+\alpha$. And as you have never thought about α you also do not know that you are ignorant or wrong about α , i.e. $\neg K_+\neg K_+\alpha$. But then, (6) tells us, you are convinced that α is true! For everything beyond your ken or interests epistemic **S4.4** for K_+ commits you to a corresponding conviction!

(S4.4) for K_+ thus should be rejected. The moral of our considerations above is: The concept of strong knowledge, K_+ , is not compatible with either unconditioned or conditioned principles of negative introspection for knowledge (i.e. the operator “ K_+ ”).

The logic of conviction incorporates a principle of negative introspection, (C4). Apart from the usual criticisms of negative introspection it does not seem to spell problems in the logic of conviction as does negative introspection in the logic of strong knowledge. But this appearance may be deceiving! The logic of conviction **C**, consisting of (C3), (C4), (C2) [the (K) axiom for the operator “**C**”], a necessitation rule (RC) for the operator “**C**” [$\vdash\alpha \Rightarrow \vdash C\alpha$], and a consistency principle corresponding to the modal axiom (D):

$$(C1) \quad C\alpha \supset \neg C\neg\alpha$$

²⁸ As “ $K_+\alpha$ ” implies by (S4) “ $K_+K_+\alpha$ ” and thus “ $K_+\neg K_+\alpha$ ” would give a contradiction by the (K) axiom.

if *combined* with the conception of strong knowledge, i.e. (K_+) , *entails* the other **S4.4** axioms for “ K_+ ” as theorems! As also **S4.4** for “ K_+ ” in combination with (6) as a definition of “ C ” entails the logic of conviction, the two logics are provably equivalent.

The direction of the logic of conviction entailing **S4.4** for K_+ is devastating. [Therefore we look only at the proof of this direction of the equivalence.] Since we have rejected (S4.4) we have to reject some of the axioms of the logic of conviction **C** if we want to keep the strong concept of knowledge!

Which of the axioms of **C** have to be given up? Let us look at the problematic direction of the equivalence proof: Axiom (T) follows immediately from (K_+) , added to **C**. (RK_+) follows immediately from (RC), the (K) axiom for “ K_+ ” follows from (C2) and propositional logic. (S4) follows from (C2) and (C3) as we have argued already above. The crucial part of the proof consists in proving (S4.4).

This proof²⁹ depends, of course, on (C4), *negative introspection*. Therefore, if we cannot accept (S4.4) and (6), even if we otherwise want to assume a strong conception of knowledge as true conviction, we have to reject negative introspection for conviction *as well*, i.e. give up (C4).

As the other axioms of the strong conception of knowledge can be derived without it we lose only the incriminated axiom (S4.4). If (C4) does not hold, and we drop the accessibility condition that the belief worlds used in modelling someone’s beliefs exhibit a Euclidian structure, we can have “ $\neg C\alpha \wedge \neg C\neg C\alpha$ ” and thus can invalidate (S4.4). As the modal logic consisting of **PC**

29 <i>Proof:</i>		
1.<1>	α	PREM
2.<2>	$\neg K_+\alpha$	PREM
3.<2>	$\neg C\alpha \vee \neg\alpha$	(K_+) , 2
4.<1,2>	$\neg C\alpha$	$(\vee E)$, 1, 3
5.<1,2>	$C\neg C\alpha$	$(C4)$, 4 [<i>negative introspection</i>]
6.<1,2>	$C(\neg C\alpha \vee \neg\alpha)$	(RC) , 5
7.<1,2>	$C(\neg K_+\alpha)$	(K_+) , 6
8.<1,2>	$K_+\neg K_+\alpha$	(K_+) , 2, 6
9.<>	$\alpha \supset (\neg K_+\alpha \supset K_+\neg K_+\alpha)$	$(\supset I)$, 1, 2, 8 ■

$\cup \{(C1), (C2), (C3), (RC)\}$ – or respectively $\mathbf{PC} \cup \{(D),(K),(S4),(N)\}$ – is sound, we are assured that there is no proof for (S4.4) in it.³⁰

Negative introspection thus is not only a very strong assumption implausible for human reasoners. Negative introspection can not be accepted for a strong conception of knowledge. And because of that it can also not be accepted for the logic of conviction if we stick to the strong conception of knowledge.

The argument does not go through using knowledge as true belief, i.e

$$(K) \quad Kp \supset p \wedge Bp$$

as the conjunction or detachment principles valid for “C” do not hold for “B”.

But nonetheless negative introspection should be rejected, since knowledge as true belief should not exclude knowledge in case of strong beliefs (i.e. convictions), and since the argument referring to the general decision procedure using a Turing machine with negative introspection still stands.

§20 Scepticism

The intelligibility of the sceptical *questions* has to be kept apart from the intelligibility of sceptical *claims*. Given our trust in our cognitive faculties we do not believe that we did not exist 5 minutes ago. Nonetheless we understand the sceptical question how we know this. We need not care about it as long as that question is not accompanied by justified doubt that we existed 5 minutes ago – it has not. We understand it precisely because we interpret it assuming along the while that we can trust our intellectual faculties. The sceptical *claim* that we did not exist 5 minutes ago, however, dissolves itself, as its constituent terms would have lost their meaning if we had not existed 5 minutes ago. We cannot believe the sceptical claim. To understand such elaborated claims presupposes understanding and believing a whole web of expressions and beliefs so that our very ability to assert or understand the claim undermines its chances to be justified or being believed at all. We cannot force us either to believe something or to doubt something. Scepticism therefore is more of a pose than a serious attitude, an option not available for us as being involved in our every day affairs (as much belaboured by Wittgenstein in *On Certainty*). Further on, beliefs are not isolated attitudes, but interwoven with our aspirations and aims. When we come to believe something we immediately relate it to the role of the assumed facts in thwarting or fostering our plans. Thus acquiring a general sceptical believe had to result in an incapability for action – this being the reason we never encounter a real sceptic.

Therefore, I consider the preoccupation of epistemology with scepticism a blind alley. *Transcendental arguments* are often seen as a reply to radical

³⁰ The system can also be shown to be complete with respect to serial, transitive frames. It is sometimes used also in Deontic Logic, then called “OS4”, as the operator “O” (for “it ought to be the case that”) does not obey (T) as well; cf. (Åqvist 1987).

forms of scepticism or to ‘a sceptic’. Replying to ‘a sceptic’ seems, to me, to be a waste of time, for the reasons mentioned and some more:

- (i) As soon as a sceptic makes some positive claim denying α he finds himself much in the same position assuming the possibility of knowledge or language as the philosophers under attack. Thus he can be easily refuted *ad hominem*.
- (ii) If to circumvent this self-refutation the sceptic is considered to ‘merely pose a question’ one may ask what that question is. If it has the form “Could it not be that ...” the meaning of “could” is far from clear. It certainly cannot be presupposed to be understood in terms of possible worlds, models or the like, on pains of falling back to point 1. Whether a vague sense of imagining other ‘possibilities’ should be taken seriously cannot be taken for granted. I don’t take it for granted. If the question has the form “Are you sure that...” one may well ask why one should answer such a question once one *has* given reasons for one’s views. A reasonable doubt falls back to point 1.
- (iii) Alternative theories should not be subsumed under the label “scepticism” to avoid confusing the issue of scepticism. So someone who doubts whether there is something like meaning in the sense of theory T_1 may better be called a “critic” as long as she does not doubt meaning in general.
- (iv) Being even a modest realist one assumes that there is or at least may be a gap between our representations, our cognitive faculties with their abilities and the complete structure of reality. To exclude the possibility that reality is – in some vague sense – greater than our cognition would be giving up realism and switching to some form of idealism in which the limits of our cognition define what there can be. So a ‘scepticism’ belabouring this gap should not be considered scepticism at all, as long as no specific limit of our cognition is mentioned, which would lead back to points 1 or 3. That there are some parts of reality beyond our abilities does, further on, not exclude that in some parts our abilities are quite appropriate.
- (v) Arguments on scepticism often start like “suppose a sceptic”. One wonders whether there really is or has been anyone claiming what ‘the sceptic’ is said to claim or ask. That no non-fictional sceptic can be presented is an observation relevant to the very issue of scepticism. If someone can really find somebody who claims to doubt the existence of things or other people he should ask him for his wallet, his car and maybe suggest to him to jump from the next bridge. Any non compliance with these suggestions is relevant to the very issue of the existence of sceptics.

The whole debate around the corresponding ‘failure’ of transcendental arguments (cf. Stern 1999) seems to be misguided. The aim of transcendental ar-

guments thus does not lay in refuting sceptics but in *delineating analytic dependencies* between concepts or assumptions. In seeing that α is a condition for the possibility of γ we recognize a conceptual connection within our conceptual scheme. Phrasing this discovery as “If a sceptic were to doubt γ she could not believe α .” adds nothing to it.

Successful arguments sometimes show that some premises entail a conclusion, sometimes the premises make the conclusion plausible. Sometimes a premise of normality (like “nothing is different in other situations”) or exhaustiveness (like “and these options considered are all the options there are”) has to be added to make an argument sound. Such premises may turn out wrong. So may be the way of science and scientific progress. As long as no reasonable doubt has been presented, however, we are justified in seeing these arguments as establishing their conclusion. They may even stand as they are for all time to come.

To require stricter standards for arguments has to be argued for concisely itself. I have not seen such arguments. Hinting at such an ultimate justification does not suffice. Recent attempts for ultimate justification in ‘apodictic evidence’ (in some period of Husserlian phenomenology) or ‘reflexive ultimate justification’ (in Apel and Kuhlmann’s [1985] *transcendental pragmatics*) are less than precisely worked out. The only point of raising the standards of justification and argument seems to be to keep some ‘sceptic’ in business. Analytic epistemology need not outdo all sciences and argumentations in its rigour. Formal (re-)constructions, meta-logic and conceptual analysis are useful and difficult enough.

§21 Perceptual Knowledge

In case of perception we have scientific reasons or world knowledge enough to consider it a generally reliable connection to our surroundings (i.e. by tracing counterfactually the presence and absence of some features). So, in case we form a perceptual belief (supposedly derivatively on processing perceptual information and accompanying a conscious state presenting a perceptual scene in at least one perceptual modality) we generally have perceptual *knowledge*. Perceptual beliefs found our empirical beliefs, and perceptual knowledge founds our empirical knowledge in the weak sense that procedures of justification *usually* stop there. Our *trust* in our perceptual beliefs depends on our presupposed background assumption that perceptions generally are reliable, giving rise to reliable perceptual beliefs, and thus to perceptual knowledge. Without this trust we cannot stop our inquiries at this point (cf. Sellars 1963: 164-70), but this background assumption can be challenged under special circumstances (like the presence of deceptive devices like mirrors or personal intoxication). Absent these special circumstances and thus in the light of the contextual standards of justification, outlined above, unchallenged perceptual beliefs

and observation sentences (i.e. those not confronted with justified doubt) are the backbone of empirical knowledge.

In the perception we perceive its object (e.g. the backyard) and not the perception itself: “I hear a scratching in the kitchen” is an example of the primary use of perception reports [method 2], in contrast to “I have the impression of some ringing in my ears”, which reflectively detaches from a direct perceptual object description to a remark on occurring perceptual or perception like states (i.e. states which involve a phenomenal state or quality). The non-primary usage will be indicated by operators or qualifiers like “it seems that ...” or “... the impression ...”.

§22 Genuine Religious Language?

Assertions made by religions like “Jesus is the son of God” are supposed to be true. The believer takes it to be a fact that Jesus is the son of God. Was religion only a way of instructing to act by moral rule (say by the Ten Commandments), it could be considered as a part of moral discourse. Religion, however, wants to be more than an ethic, and the ethic given is thought to be grounded in religious *facts* (e.g., that it is a fact that God made the world, that God gave us the commandments – etc.). Religious speech contains speech acts like commanding, praying, exclamations or singing. OLP might look at the patterns of usage of such expressions. Cultic acts are distinguished from playing or theatre, however, by claiming that the objects which are addressed in a song or a prayer are really existent. Making assertions, therefore, is fundamental to religious language. We can therefore ask for the conditions of assertability of religious descriptive sentences [method 8]. For a complete analysis of religious language a mere description of non-assertoric religious language does not suffice.

Suppose then that religious claims could make sense. But they really make sense only if the terms employed in these assertions have a well defined meaning. The meaning of a sentence is well defined only if there are conditions in which the application of that very sentence in distinction to other sentences will be considered right and conditions under which its application will be considered wrong. An expression generally can be considered *well defined* in its meaning if its semantic distinctness supervenes on the structures of the world (structural difference between round and square objects in the world provide the basis for distinguishing applying “round” or “square”). ‘generally’ if there was an alternative for religious expressions. Such an alternative is not to be seen [method 11]. Using a well defined expression employs criteria of use. A theory of religious language use cannot neglect this, since we can ask why the expression “Odin” was used instead of “Thor” if these have definite meaning. Even if the rules of use are determined in part by the religious belief system itself the expressions are always connected to ordinary expressions (e.g. “has only one eye”), which have definite criteria of applicability. On

pains of making religious terms void of content this connection cannot be completely severed. The only consistent act for somebody who claims the inexpressibility of religious belief to counter this reasoning is to remain silent. A mere announcement that the expressions occurring in religious statements are not used in their ordinary understanding gets into even more trouble: On pains of not being understood it has to be possible to translate metaphors and hints into more precise terms; if this is not possible, one may ask whether the metaphors or the hints are meaningfully employed at all. Religious statements cannot be merely unusual. If the expression "God" can be subsumed under different and non-arbitrary predicates, the use of these predicates and not others has to be based on facts concerning the properties of God. It is not sufficient to call religious speech 'metaphorical'. In metaphorical speech a transfer takes place (expressions are used to convey something that can be expressed by them because of a partial, maybe abstract, structural correspondence between their usual conditions of use and the intended area or object of reference). In any metaphorical use, however, there are before applying a metaphor descriptions which are used non-metaphorically. If you express something that was described before by using a metaphor, still there was a (partial) non-metaphorical description before. This applies also if something that is described at the moment (say a perceived object) has not been put into words before. So for example in case of seeing you might have „This is darker than its background". In the case of hearing you might have "This is louder than a car". The use of metaphors depends on such expressible distinctions between ordinary properties. So an ordinary description, even if only partial, precedes a religious description. Objects of religious descriptions have to be describable non-metaphorically if they are to be described metaphorically or 'analogously'. Metaphorical language use, therefore, cannot defend the claim that religious terms refer to something if it is claimed at the same time that they do not refer by ordinary language use. By being non-describable in non-metaphorical speech an object just is non-describable simpliciter. The only alternative is that the perceived, so far undescribed object is perceived in a mode of perception in which partial non-metaphorical descriptions are not at hand. Religious experience might be like that. An object given in a religious experience, understood thus, could not be described by ordinary usage in any way. Its expressibility would depend on a completely different way of referring to objects. The thesis that religious terms have a definite meaning would not only imply that there has to be some religious experience, but would also require that there is a distinct mode of expressing these experience that does not work like ordinary perceptual reports. Morphological rules can give us expressions (like „inexpressible“, „all-knowing") which have a definite meaning because of the definite meaning of their parts, even though we cannot make out whether their conditions of use are present. Some religious terms can be embedded thus into our ordinary system of meanings. They enter into logical relations with other sentences including observational sentences. Sentences con-

taining them have meaning and can be used to make statements that are true or false.

§23 Another Basis for Religious Language?

Somebody who wants to defend religious language could reply thus to the preceding paragraph:

Asking for the meaning of religious terms presupposed our ordinary conception of meaning and reference (applied paradigmatically to physical objects). Our non-logical vocabulary refers first to physical objects and then to mental events. With respect to these entities conditions of use of a term can be specified, and it is a fact of the matter which can be decided intersubjectively whether these conditions are given or not. Talking about physical entities is the paradigmatic case of introducing expressions. Is it proper to criticise religious language from this perspective?

A justification of that criticism is that our conception of meaningful language is founded in this paradigm. Meanings have to be intersubjective. The conditions of use of ordinary terms guarantee that. Furthermore religious language claims truth. Claiming truth involves intersubjective justifiability. And intersubjective reference is a way to do this. And, as we have seen, religious language contains ordinary expressions which are defined by their normal rules of use.

Once again a reply by the proponent of genuine religious language is possible: What if there were other ways of speaking? To criticise them on the standard case would be a *petitio*. All that is shown by non-fulfilment of the standards of ordinary reference is that neither we talk about objects in space and time nor about mental events, i.e. we talk about a realm of objects *sui generis*. The claims of religious language involve truth, but a connection between meaning and physical referents is needed to require that we have to talk in the ordinary way in religion. Meaning might be founded in different ways. And besides the ordinary expressions in religious talk there are the genuine religious terms. Using ordinary terms cannot be avoided because of the volatile nature of the religious realm and the need to introduce it by using analogies.

This reply needs a foundation in a theory what it means to talk religiously in the narrow sense. Suppose there is a realm of reality which is not the realm of physical objects. It could scarcely be described in ordinary terms. Using them for this job would convey the impression of vagueness or improper usage. This realm needed a new vocabulary. We can think of this vocabulary as being similar to our ordinary categories, so we might assume that the new realm contains objects and properties, although they might lack the stable structure we know from physical objects. This might be the very definition of this realm. So the vague language would correspond to the volatile objects residing there. Descriptions would have to be interpreted as being associative and as

trying to evoke in the listener an access to the objects referred to. Since somebody else could thus understand my religious talk it would not be merely subjective.

So a theory might run, but I take it to be extremely difficult to deliver on such promises of a theory. The mere conceptual possibility, if we grant that, does not compensate for the missing systematicity of such talk [cf. method 11] exhibited so far.

§24 Revelation and Religious Language

Though even if not all religious terms are meaningful, there might be some that have a well defined meaning. If transcendent objects are not only transcendent but spend some of their time in our universe religious objects can be talked about with ordinary vocabulary.

Claiming the existence of a revelation is another characteristic feature of religious talk in many religions. As a move in the religious language games it is analogous to going back to the data of observation in the sciences. Nevertheless revelation cannot be reproduced intersubjectively. There is a fundamental problem with revelation: since there are too many candidates for revelations the source of the revealed message has to be established as being (religiously) reliable beyond doubt; now this means either we get a regress of revealed reliability of the revelation's source or the contents of the revelation guarantee its authenticity making it thus superfluous for them to be revealed (instead of just spoken) in the first place. So distinguishing one source of revelation is part of faith itself. Revelation itself cannot justify belief to the unbeliever. A coherent faith might, however, be erected upon the initial assumption.

A particular solution can be given within those religions in which God speaks to us. He is speaking in *our* language. So his words are understandable to us. We recognise him – depending on the particular faith – as God either because of his words (as the Quran claims) or because of his miraculous deeds accompanying his revelation (as in the case of the incarnation in Jesus Christ). In this case religious language is founded by God. He neither needs religious experience nor a peculiar kind of language – he just is in the transcendent realm. From the moment, however, we start interpreting his revelation and try to describe God in his properties, especially when reporting of a religious experience of him, the old difficulties reappear. We can talk about Jesus, inasmuch as he is human, in an ordinary way, but not of the son of God or the Holy Spirit – and that is the crucial part of our description. So we need besides or instead of the faculty to express some genuine religious experience a special faculty – in the Christian faith provided by the Holy Spirit – to understand the revelation. Obviously this approach does still presuppose a lot. In both theories (expressing a genuine religious experience vs. being able to understand God speaking to us) accepting religious language remains part of faith itself. Referring to the Holy Spirit as securing the proper understanding of a revealed

message will not convince a non-believer. Claiming that only the chosen ones can understand the message leads straight into esoteric isolation. In case of the assumption of genuine realm, a genuine experience and a genuine language we need a distinction to hallucination or muddled talk. The religions of the book can at least provide a coherent system of religion which does not make that many presuppositions. Christianity diminishes the role religious experience and expressing that experience plays. Maybe we do not need any religious experience of a problematic sort at all. This can be seen by drawing a distinction within the notion of religious experience:

Religious experience can be introduced as experience of religious objects (i.e. the experience is characterised *referentially*). This need not require any new mode of experience. When Thor splits a tree and we see the split tree we have an experience of Thor's deeds independent from the fact whether we describe it thus or not. Religious experience can, secondly, be introduced by a mode of experience. According to this it would be evident for the experiencing person that she has such an experience because she finds herself in state peculiar mental state. Some religions (e.g. those speaking of 'enlightenment' or 'satori') require for their foundation an account of religious experience of the second sort, which seems to be difficult to provide. Religion in general does not need such an account. Religions based on revelation in language (like Christianity or Islam) refer to a religious entity (God) because that entity is talking to us in *our* – maybe somewhat extended – language. The authorisation of revelation may happen by miracles. Miracles are exceptions to the laws of nature, but they are *not* exception to our perceptual faculties. People just saw Lazarus coming back from his grave. So a religious experience of the second, more problematic kind need not be claimed here. So even if there is no such religious experience and thus much of religious language cannot be founded on it, there are still religious claims assertible.

5.4 Conceptual Analysis in Analytic Philosophy

§25 Analysis and Analytic Philosophy

Analysis can be understood as referring either to the process or to the result of an inquiry. In philosophy, especially OLP, the result may be seen to consist in stating, explicating or repeating (with therapeutic intent) analytic or 'grammatical' sentences.

Either there are analytic sentences or there aren't.

If there aren't analytic sentences the results of philosophy aren't vindicated by some special status (i.e. being 'analytic', 'a priori', 'unrefutable' etc.), but by being methodologically established by procedures of 'analysis' which fulfil the standards of scientific rigour, and which involve methods (like discussing 'cases' and using tools of formal languages) characteristic of philosophical argumentation. Philosophical argumentation is non-empirical in that it relies on scientific data, in case it relies on scientific data at all, which have been gathered by empirical sciences, not by philosophy itself. Non-scientific data philosophy may rely upon are, for instance, some of the problematic cases discussed. They usually describe a common phenomenon and can trust the general (folk) experience of the reader to know the phenomenon (e.g. that somebody has an accidentally true belief). Analysis in that sense then reveals the structure of a problem or a topic in question. We learn something or see the issue clearer after the analysis. We may give up some of our prior opinions and derive at a clearer description of some part of the world (including ourselves and our cognition). Analysis in this sense ideally arrives at a theory expressing a more adequate, more comprehensive understanding of some part of the world. This may involve means of formal analysis (like inventing new forms of expression or re-regulating prior forms of expression, including – revisionary – explicating ordinary language). Philosophy then can be distinguished from other sciences not by the status of its results, but by (1) a tradition of topics typically not dealt with in the special sciences, or (2) a meta-scientific choice of topics (e.g. focussing on the varieties of a general concept of 'justification', presupposed in the 'special' sciences). Often this type of philosophy will be continuous with the other sciences or at least with their reflection on their own conceptual or methodological foundations. Some of its topics (*inter alia* theories of concepts) will even be identical with topics dealt with in the cognitive sciences, philosophical and linguistic or psychological theories being hardly distinguishable.

If there are analytic sentences much of what has just been said about philosophy can still be true. The pressing next question, however, asks whether these analytic sentences are substantial or not, where “All cats are mammals” may be an example of an unsubstantial and “Knowledge is true belief” an example of a substantial analytic sentence (supposing that they are analytic). Philosophy aims at substantial insights. There may well be substantial analytic sen-

tences, *even* if they deliver not a complete decomposition of a concept (e.g. knowledge may be essentially more or less than true belief). That these sentences are analytic does not exclude their substantial content, because language ideally captures substantial insights about the world in the meaning or the semantic links of its expression of our conception of the world. Therein rests the authority of language. As not all our insights – even into fundamental questions – had time to be made part of the (partial) definition of a concept or a word, a restriction to state *only* analytic sentences as results of philosophical inquiry will be a curtailment of available insight. The whole issue of the status of the sentences expressing philosophical insights has been overrated and discussed out of proportion. As there is not one single philosophical method and philosophy draws on the results of different sciences or folk observations or common sense or common knowledge, so the status of the results of philosophical inquiry will differ.

I understand analytic epistemology as being part of the wider study of cognition (in the cognitive sciences). It deals not with the actual details and features of the human psyche or human brain, but tries to outline some necessary features (conditions and rules) of having thoughts and beliefs at all, of being able to judge at all. The value of its analysis has to be assessed not only by confronting them with other philosophical theories, but also by confronting them with our empirical knowledge about the workings and limitations of human cognition. Whether one wants to call it “*a priori*” is of no importance at all, once its peculiar methods are set out and kept apart from other approaches in science. Especially any pretence of “*a priori*” meaning “unrevisable” has to be dropped, having done philosophy not a single favour.

Following the tradition of defining a science by (i) its topic or area of application, its ‘object’ and (ii) its method, we can sum up the discussion with the following mouthful of a definition of philosophy. Philosophy is *universal* in its topic and methods: its topics include everything (including even supposed entities and *itself*), it refers freely to any established methods of the sciences and expands them by the reflexive methods of model building and expansion or reform of language.

Analytic philosophy may distinguish itself from other approaches in philosophy by a canon of methods developed and used in the analytic tradition (of either OLP or ILP). The distinction itself can be worthwhile if enough of these methods achieve higher scientific standards than those practised in other philosophical camps.

§26 Analysing the Fundamentals of our Conceptual Framework

The proper idea of a philosophical theory of our fundamental conceptual framework focuses on questions before and beyond empirical science.

These questions concern – *inter alia* – those of setting up at least the core of the linguistic framework of the kind of study in question. The core of the lin-

guistic framework is not concerned with the definition of theoretical concepts of the science in question, but with questions like the expressive power of the linguistic framework needed (e.g. do we need higher order quantification in that area or do we need a syntactic/semantic category of processes) and the arsenal of inferential methods (e.g. do we have to be able to have probability assignments and procedures of conditional updating). A couple of these questions are somewhat continuous with foundational studies in a field of science. Some concepts and questions, however, are so general that they are not treated even in foundational studies of individual sciences. Questions about the nature of *truth* and *sufficient justification* or the comparison of seemingly equally coherent theories (including an outline of what *coherence* consists in) belong in this category. Even in those cases where there is some overlap to foundational studies (say in arguing for basing temporal ontology on points or on intervals) the scientists are now engaged not in typically empirical investigations, but in a typically philosophical reflection on the proper construction of a linguistic framework.

Our concept of language involves unity and universality. There has to be a set of properties defining what a language is. These properties are preserved in change or translation, they are exploited to establish correspondences. Elucidating these properties and making them explicit from our intuitive understanding of natural language is the traditional understanding of a universal philosophy of language, its proper object of study being the ways we linguistically communicate.

There is a multitude of discourse structures. On the one hand we may distinguish discourse types like scientific discourse – the one type typically in focus – and aesthetic discourse, which obviously cannot aim at intersubjectively shared truth in the sense of scientific knowledge. On the other hand we can direct our investigations not only at the structure of sentences and statements, but also on the illocutionary acts involved in making statements or the presuppositions and implicatures in a situation of cooperative communication.

Conceptual analysis aims at such elucidations. It presupposes that there is some semantic structure to the framework. Arguments of conceivability, transcendental arguments [like the Fregean argument in chapter 2], and model building all aim at tracing the semantic roles and connections in this framework. Since there is this semantics and conceptual analysis traces its workings its essential results are *analytic sentences*, one may even say that successful elucidation of the framework reveals the *synthetic a priori* principles at work in our mental faculties. Again, nothing depends on these labels, which have had their share of philosophical bad press. Notwithstanding this conflict with current tastes the status of the principles explicated is beyond those of mere empirical generalizations. Even naturalists should not ignore the existence of dictionaries and the conceptual links they try to capture.

Conceptual analysis itself has had its share of philosophical bad press. In part – as with phenomenology – this might have been because of the sometimes subjective quality of its findings or musings. Extended empirical investigations and technical research certainly outstrip the means of a (couple of) researchers. Conceptual analysis seems to provide the conditions of possibility from an easy-chair perspective. Nonetheless, if there are innate concepts they are *a priori* from the individual speaker's point of view. Conceptual analysis then should have a chance of succeeding (with respect to basic concepts). Conceptual analysis has to play its part, the seeming subjectivity of some of its findings will be checked in a broad reflective equilibrium with other findings and other models.

Ist Hegels Dialektik reine Begriffsanalyse?

§1 Vorbemerkung

Ausgangspunkt des hier vorgestellten Interpretationsansatzes ist die Annahme, dass zumindest Ansatz und Anliegen von Hegels Dialektik explizierbar sind, selbst wenn einzelne Übergänge in der *Wissenschaft der Logik* und die Behandlung einzelner Subthemen (wie der 'Widerspruch') völlig falsch sind. „Explizierbar“ heißt dabei: erläuterungsfähig auf dem Stand systematischer philosophischer Erläuterungen, somit jenseits einer bloß reproduzierenden Hegel-Nacherzählung.

Das Interesse an einem Verständnis der *Wissenschaft der Logik* mag sich dabei aus ganz verschiedenen Quellen speisen: Für diejenigen, die Hegels sonstige Philosophie (des Rechtes, oder der Geschichte, oder ...) schätzen, muss sich irgendwann die Frage nach der genaueren Explikation der dialektischen Methode stellen, deren Anwendung Hegel dort behauptet und zu deren Erläuterung er auf das Buch *Wissenschaft der Logik* verweist.³¹ Für diejenigen, die in der ein oder anderen Form an Kants Transzendentalphilosophie anknüpfen (wollen), stellen Kants unkritische Orientierung an der klassischen Logik und der newtonschen Physik sowie sein Transzendentaler Idealismus Hemmnisse dar, und Hegel präsentiert sich als Kant-Kritiker, der sowohl an Kants transzendentalphilosophisches Programm anknüpfen will und dabei zugleich das enge Logikverständnis der Kritik der reinen Vernunft wie den Transzendentalen Idealismus hinter sich lässt. Für die gelungene Ausführung des kantischen Programms verweist Hegel auf die *Wissenschaft der Logik*. Für diejenigen die in Marx einen Nachfolger Hegels sehen, der sich auch der Dialektik bedient, könnte es interessant sein, die Ursprünge der Dialektik von Marx oder des 'Dialektischen Materialismus' in Hegel zu suchen.

Die folgenden Überlegungen orientieren sich zum einen an der zweitgenannten Motivation. Hegel wird aus der Warte einer (analytisch gewendeten) Transzendentalphilosophie betrachtet. Zum anderen wird Hegels Dialektik aus der Warte des in den vorangegangenen Kapiteln entwickelten Verständnisses von ‚Begriffsanalyse‘ betrachtet. Hegels Dialektik betrifft Begriffe, die irgendwie auseinandergelegt und auf einander bezogen werden. Es scheint also *prima facie* eine begriffslogische Methode vorzuliegen.

Das und die Überlegungen im Einzelnen mögen Hegels Selbstverständnis und Hegels Ausführungen in Vielem nicht gerecht werden. Es wäre m.E. allerdings schon

31 Zitiert wird hier nach der Suhrkamp-Ausgabe: „I“ und „II“ beziehen sich auf die beiden Bände der *Wissenschaft der Logik* (WdL), „E“ auf die Logik in der *Enzyklopädie der Philosophischen Wissenschaften*.

viel gewonnen, wenn es überhaupt einen kohärenten Zugriff auf die Wissenschaft der Logik gibt, der sich teilweise mit Hegels Text belegen lässt.

§2 Kant und Hegels verschiedene Perspektive auf die Widersprüche der reinen Vernunft

Kants Dialektik ist Teil seines Gesamtplanes: in der theoretischen Philosophie wird ein Platz geschaffen für die Freiheit. Kant will die total determinierte Welt einer newtonschen Physik versöhnen mit einer Theorie der Freiheit, die benötigt wird für die moralische Verantwortlichkeit des autonomen Subjekts sowie die bloße Möglichkeit des moralischen Sollens. Kant erkennt die Unhaltbarkeit eines Kompatibilismus und sieht durch den Inkompatibilismus die siegreiche neue (newtonsche) Physik bedroht. Dieser will er nicht widersprechen. Er vereinigt Inkompatibilismus und Determiniertheit der Physik, indem er die Freiheit in einen anderen Bereich – eine zweite Welt (die 'noumenale') – verweist. Das ist der entscheidende Konstruktionsansatz für die Transzendente Dialektik. Die theoretische Philosophie in Form der Transzendentalen Dialektik etabliert die entsprechende Weltenteilung. Die Transzendente Analytik zeigt, wie wir die Erscheinungswelt konstruieren. Sie rechtfertigt zugleich die empirischen Wissenschaften. Die Transzendente Dialektik zeigt, dass die Unterscheidung von Erscheinungswelt und noumenaler Welt nicht allein ein Postulat von Kants Freiheitstheorie ist. So bliebe immer die Option, die Theorie der Freiheit zurückzuweisen.³² Die Transzendente Dialektik zeigt, dass man diese Unterscheidung machen muss – um der Konsistenz der reinen Vernunft willen. Die praktische Philosophie schließt sich so 'nur' an eine schon im Theoretischen gemachte Unterscheidung an.

Für Kant zeigen die Antinomien, dass es einer Unterscheidung zwischen Ding(en) an sich und Erscheinungen bedarf, da Dinge nicht widersprüchlich sein können, die mutmaßlichen Widersprüche nur die Erscheinungen betreffen. Damit wird auch der Begriff des 'Ding an sich' gerechtfertigt, unangesehen, dass sich die Rede vom 'Ding an sich' in Probleme der Ausdrückbarkeit verstrickt.

Für Kant sind daher die Konstruktionen der Beweise für eine These und die entsprechende Anti-These in der Dialektik von entscheidender Bedeutung. Nur wenn sie gelingen, ist die Erforderlichkeit der Erscheinung/'Ding an sich'-Unterscheidung nachgewiesen.

So besehen kann man sagen: Ist man Kompatibilist oder Determinist, falls diese Positionen haltbar sind, oder Inkompatibilist, der sich um die (zu) allgemeine Theorie einer vollständigen Determiniertheit der physischen Welt/Wirklichkeit nicht kümmert, oder eine quantenphysikalischer Indeterminist bezüglich der physischen Welt/Wirklichkeit, dann erübrigt sich das entscheidende Motiv für die kantische Dialektik. Ob ihre jeweiligen Argumente für Thesen und Gegenthesen gelingen oder versagen, spielt dann keine große Rolle mehr. Was dann an der Transzendentalen Dialektik noch interessant bleibt, ist die Konzeption von Vernunftideen, welche

32 Man bedenke, die Deduktion der Freiheit in der *Kritik der praktischen Vernunft* lag hier noch nicht vor.

analoge Rollen innehaben wie die Kategorien des Verstandes. Auch muss man unterscheiden zwischen Kants – zu wenig beachteter – Idee, die Trugschlüsse der Vernunft aus dem konstitutiven Gebrauch von Heuristiken zu verstehen und Kants misslingender Konstruktion von allerlei (Pseudo-)Antinomien, welche eine zweifelhafte erkenntnistheoretische Position (den Transzendentalen Idealismus) rechtfertigen. Kants Grundidee könnte z.B. Anwendung finden in einer Zurückweisung der Idee einer determinierten kausalen Geschlossenheit der Wirklichkeit als Missbrauch der wissenschaftlichen Heuristik, nach zureichenden Kausalerklärungen zumindest zu suchen.

Hegel macht die ganze theoretische Strategie, die Kant mit seiner Dialektik verfolgt, nicht mit, da er die Annahme eines uns verschlossenen 'Dinges an sich' ablehnt. Dadurch werden die Antinomien dann zu Beweisen von realen Widersprüchen in der Sache (d.h. sie sind nicht mehr 'äußerlich': sei es in der Erscheinung oder unserer begreifenden Reflexion).

§3 Hegels Kant-Kritik

Hegel übernimmt nicht Kants Unterscheidung von Erscheinung und 'Ding an sich'. Dabei weist auch Hegel schon auf den idealistischen Trugschluss hin: Allein daraus, dass bestimmte Strukturen (seien es Kategorien oder 'Anschauungsformen') zum Gerüst unseres kognitiven Zugangs zur Wirklichkeit gehören, folgt nicht, dass die Wirklichselbt nicht diese Strukturen aufweist; Konstruktion kann Re-Konstruktion sein. Dies gilt umsomehr, wenn man eine Theorie der Kognition heute in eine Theorie ihrer natürlichen Geschichte in der natürlichen Anpassung einbettet.

Ob nun schon die Kategorien (wie z.B. Einheit, Ursache und Wirkung usw.) dem Denken als solchen zukommen, so folgt daraus doch keineswegs, daß dieselben deshalb bloß ein Unsriges und nicht auch Bestimmungen der Gegenstände selbst wären. [E, 119]

„Nach Kant ist dagegen dasjenige, was wir denken, falsch darum, weil *wir* es denken.“ [E, 146]

Die Transzendentalphilosophie muss also keinen Idealismus mit sich bringen.

Hegel sieht daher – hier treffen sich Idealismus und ein starker Realismus – auch keine Erforderlichkeit Metaphysik und Logik zu trennen: es geht ihm um die eine Untersuchung der allgemeinsten Kategorien. Es geht um den Gehalt dieser Bestimmungen, und sonst nichts: „Auf jenen Unterschied von Subjektivität und Objektivität kommt also überhaupt nichts an, sondern der Inhalt ist es, worauf es ankommt.“ [E, 119] Damit ist die Wissenschaft der Logik „die eigentliche Metaphysik oder reine spekulative Philosophie“ [I, 16].

Hegel will in zwei Hinsichten mindestens über Kant hinaus. Zum ersten wirft er Kant vor, dass dieser zwar einige Kategorien (reine Bestimmungen³³ des Denkens)

33 Ich verwende „Bestimmungen“ als allgemeinen Ausdruck für reine Begriffe des Geistes, also für „Kategorien“ und „Ideen“ im kantischen Sinne und „Begriffe“ im Verständnis, dass nicht

thematisiert habe, diese indessen lediglich aufgegriffen und gesammelt, nicht systematisch entwickelt habe. Kant habe die Kategorien

im Subjekt gelassen, wie sie vorgefunden. ... so daß [nicht] von einer Ableitung ihrer an ihnen selbst oder auch einer Ableitung derselben als subjektiv-logischer Formen, ..., derselben die Rede war [I, 40f.].

Eine systematische Entwicklung der Bestimmungen des Verstandes/der Vernunft allein kann gewährleisten, dass alle Bestimmungen gefunden wurden. Hier würde Kant wohl auf die Deduktion der Kategorien und auch der Ideen relativ zur Urteils-tafel verweisen, die in ihrer Vollständigkeit auch die Vollständigkeit der Kategorientafel und des Inventars der transzendentalen (regulativen) Ideen der reinen Vernunft verbürge. Selbst wenn dem so wäre – Kant geht ja irrigerweise von der Adäquatheit der klassischen aristotelischen Logik aus – fehlt, für Hegel, immer noch eine Erläuterung des Zusammenhanges, den alle diese Bestimmungen untereinander besitzen.

Zum zweiten sieht Hegel Kategorien und Ideen als eingeschlossen in ein System des reinen Geistes, trennt also nicht Verstand und Vernunft, wie Kant dies tut, wobei er (vgl. §2) die kantische Dialektik gerade als Beweis des Vorkommens von Widersprüchen in der reinen Vernunft sieht, deren ausgezeichnete methodische Bedeutung Kant übersehen hat. Die kantische Unterscheidung von Verstand und Vernunft macht, für Hegel, die Vernunft letztlich zur Dienerin des Verstandes, der sich um empirische Erkenntnisse bemüht. Die Vernunft wird „darauf eingeschränkt, nur subjektive Wahrheit, nur die Erscheinung zu erkennen“ [I, 38]. Kants Theorie des transzendentalen Scheins „erkennt nicht, daß der Widerspruch eben das Erheben der Vernunft über die Beschränkungen des Verstandes und das Auflösen derselben ist“ [I, 39].

Innerhalb der Seinslogik ergibt sich eine weitere Differenz, die auf Hegels Kritik an Kants Transzendentaler Dialektik zurückweist. Kant steht in der vormodernen Tradition, dass Unendliche als potential Unendliches aufzufassen. Das Unendliche ist ein Progressus ins Unendliche. Wie schon Aristoteles und einige seiner Vorgänger

Ausdrücke gemeint sind. Zur Eindeutigkeit wird die Rede über die Bestimmungen selbst durch Verwendung von Großbuchstaben angezeigt. Man könnte auch von „Bedeutungen“ sprechen (in einem System der Bedeutungen). Jede dieser Bestimmungen hat einen Gehalt; synonym zu „Gehalt“ verwende ich auch „Inhalt“. Offensichtlich liegt dem Programm der *Wissenschaft der Logik* die Idee zugrunde, dass der Gehalt von Bestimmungen durch ihre (logische) *Rolle* in einem System der Bestimmungen *vollständig* erfasst werden kann (s.u. §4). Eine Position, die man heutzutage „inferential role semantics“ nennt. Jerry Fodor sieht – m.E. nicht zu Unrecht – in den Variationen dieser Position einen Grundfehler der gegenwärtigen Kognitionswissenschaften und Philosophie des Geistes. Hegel muss also eine vollständige Begriffsanalyse behaupten, womit deutlich über die allgemeine Annahme, dass Begriffsanalyse über analytische Beziehungen möglich ist, hinausgegangen wird. – Auch diesbezüglich geht es hier 'nur' darum, Hegels Ansatz zu verstehen, selbst wenn er von problematischen Annahmen zehrt. Eine *inferential role semantics* bzw. eine Festlegung auf komplette Begriffsanalyse scheinen dabei nicht die schwersten Hypothesen zu sein, schwerer wiegt die Idee des direkten (mental)en Zugriffs auf Bestimmungen (s.u. §5). Demgegenüber kann man m.E. Hegels Anliegen in der *Wissenschaft der Logik* vom (objektiven) Idealismus – was immer das genauer sein mag – trennen.

das Unendliche als das Nichtdurchschreitbare auffassen, so konzipiert Kant das Streben nach dem Unbedingten (im regulativen Gebrauch der Ideen in der reinen Vernunft) als einen nicht abschließbaren Progressus. Die Annahme, die Reihe der Bedingungen sei mehr als 'aufgegeben' und insofern als Objekt 'gegeben' liegt an der Wurzel des 'transzendentalen Scheins' und der Trugschlüsse der reinen Vernunft. Hegel hingegen unterscheidet, nachdem er vom ETWAS aus die Bestimmung ENDLICHKEIT entfaltet hat zwischen 'schlechter' und 'wahrer' UNENDLICHKEIT. Die schlechte UNENDLICHKEIT ist der Progressus: das Endliche wird überstiegen, aber gefasst wird so nur ein erweitertes Endliches, das wieder überstiegen werden kann – usw. Die schlechte UNENDLICHKEIT enthält so immer noch den Verweis auf die ENDLICHKEIT. Sie tritt auf als das bloße Jenseits der ENDLICHKEIT. Damit droht jedoch die Eigenständigkeit der UNENDLICHKEIT verfehlt zu werden. Hegel scheint hier teilweise die spätere Entwicklung der Theorie des Unendlichen bei Cantor zu antizipieren. Eine der wichtigen Schritte Cantors zur Begründung der Mengenlehre, war die Einsicht, dass der Bereich, aus dem immer weiter Elemente entnommen werden, über deren Gesamtheit (etwa als Zahlen) zugleich gesprochen wird, auch selbst als Menge/Gesamtheit vorliegen muss. Das führt zum Postulat der Existenz des aktual Unendlichen.³⁴ Hegels Unterscheidung bezüglich der beiden Unendlichkeiten deutet in diese Richtung: Die wahre UNENDLICHKEIT ist das aktual Unendliche! Von ihm gilt „Es ist und ist da, präsent, gegenwärtig.“ [I. 164]. Gibt es somit das abgeschlossene Unendliche fällt ein weiterer Baustein in Kants Konstruktion der Transzendentalen Dialektik.

§4 Gegenstand und Thema der Wissenschaft der Logik

Die Logik ist für Hegel mehr als bloß formal oder bloße Propädeutik. Ihr Grundthema sind die Kategorien/Bestimmungen des reinen Geistes. Damit vereinigen sich Metaphysik und Logik. Beide befassen sich mit den Grundformen alles Seienden, einem Grundgerüst von Bestimmungen, inklusive deren Beziehungen untereinander. Die spekulative Philosophie erörtert und führt die Methode zur Erkenntnis dieses Grundgerüsts von Bestimmungen im Rahmen der richtig verstandenen Logik als Dialektik vor.

Bei Hegel selbst knüpft die Wissenschaft der Logik an die Phänomenologie des Geistes an. Sie führt diese fort bzw. setzt an der Stelle ein, zu der uns die Phänomenologie des Geistes, laut Hegel, geführt hat. Man kann diesen Standpunkt, dieses Niveau/Level der Untersuchung aber auch unabhängig vom mutmaßlichen Ergebnis der Phänomenologie des Geistes charakterisieren: Es geht um einen Standpunkt jenseits des bloßen – zumeist empirisch infizierten – Bewusstseins, einen Stand-

34 In der Standardmengenlehre ZFC tritt es als das Unendlichkeitsaxiom auf, das die Existenz mindestens einer unendlich großen Menge postuliert. Es handelt sich dabei um ein metaphysisches Existenzaxiom, das als solches schon den Logizisten wie Russell – und teilweise auch Zermelo selbst – Sorgen bereitet hat, insofern es nicht rein logische Natur sei. In ZFC, wie bei Cantor selbst, folgt aus der Existenz einer unendlichen Menge, mit dem Potenzmengenaxiom und damit *Cantors Theorem*, die Reihe der immer größeren unendlichen Mengen, die Kardinalzahlarithmetik. All das kannte Hegel natürlich nicht.

punkt des sich selbst denkenden reinen Geistes. Auf diesem Level der Untersuchung fehlt jedes äußere, empirische Beiwerk, hier thematisiert sich der reine Geist selbst.

Mit der 'Aufhebung des Bewusstseinsgegensatzes' ist zweierlei gemeint: zum einen geht es darum, dass der reine Geist den reinen Geist thematisiert, es also keinen Gegensatz zwischen dem gibt, was denkt, und dem, was Thema dieses Denkens ist, ganz anderes als im empirischen Bewusstsein; zum zweiten geht es immer auch um den methodischen Umstand, dass nicht an einen Gegenstand eine Methode herangetragen wird, sondern der Geist sich des System seiner Bestimmungen inne wird, so wie diese ihrem (objektiven) Gehalt nach auf einander bezogen sind.

Das Logische ist , ..., als ein System von Denkbestimmungen überhaupt aufzusuchen, bei welchen der Gegensatz des Subjektiven und Objektiven (in seiner gewöhnlichen Bedeutung) hinwegfällt. [E, 81]

Die Logik ist sonach als das System der einen Vernunft, als das Reich des reinen Gedankens zu fassen. [I, 44]

Die Bestimmungen des Gerüstes des reinen Geistes haben wir auf der einen Seite immer schon, denn wir besitzen reinen Geist (Vernunft). Was aber so eigentlich im Geist schon vorhanden ist ('an sich' anwesend) und im Umgehen mit einzelnen Erkenntnissen 'bekannt' ist, ist darum noch nicht erkannt. Das Grundanliegen teilt hier die Wissenschaft der Logik mit der Kritik der reinen Vernunft bzw. der Transzendentalphilosophie im Allgemeinen: das immer schon vorausgesetzte Grundgerüst des reinen Geistes, der transzendente Rahmen der reinen Vernunft soll expliziert werden. Implizit zehren wir in unserem Denken und Erkennen immer schon von diesem Begriffsgerüst, wir haben seine komplexe innere Struktur deswegen aber noch nicht explizit erfasst. Die Konstellation stimmt auch mit dem Explizitmachen von Bedeutungen in der Analytischen Philosophie überein. Hegel zielt auf die Bedeutungen/Begriffe selbst.

Dass Geschäft der Philosophie besteht nur darin, dasjenige, was rücksichtlich des Denken den Menschen von alters her gegolten, ausdrücklich zum Bewußtsein zu bringen. Die Philosophie stellt somit nichts Neues auf; was wir hier durch unsere Reflexion herausgebracht, ist schon unmittelbares Vorurteil eines jeden. [E, 79]

Die Wissenschaft der Logik bemüht sich um diese Selbstaufklärung des reinen Geistes. Hier knüpft Hegel an Kants Programm an. Es geht um „Rekonstruktion“ [I, 30].

Man könnte somit sagen: Gegenstand der Wissenschaft der Logik ist das Grundgerüst des reinen Geistes: das Gesamt der logisch/metaphysischen Bestimmungen, der transzendente Rahmen des Denkens von etwas. Das ist 'die Sache'. Die Begriffe/Bestimmungen, um deren Zusammenhang es geht, sind jeweils der Kern der Bedeutung der Ausdrücke, die verwendet werden, um den transzendentalen Rahmen darzustellen (vgl. §5). Sie sind allen Sprachen gemeinsam. Die Übergänge in der Logik erfolgen, weil die Bestimmungen so sind, wie sie sind. Der gesamt transzendente Rahmen ist implizit gegeben (schon Bestandteil unseres Geistes). Und auch die Relationen sind schon gegeben, der Pfad durch sie 'immer schon' durchschritt-

ten. In der dialektischen Bewegung tritt der immer schon vorliegende Zusammenhang der Bestimmungen explizit in unser Selbstbewusstsein. Warum die Bestimmungen – vor allem der Anfang der Wissenschaft der Logik – so sind, wie sie sind, muss gar nicht erklärt werden. Gefragt wird nach den Bedingungen der Möglichkeit (dem transzendentalen Rahmen), nicht dessen externer Genese. Mit uns liegt er immer schon vor.

§5 Das Darstellungsproblem

Schon bei der Definition der – richtig verstandenen – Logik weist Hegel auf den Unterschied von 'äußerer' und einer entgegengesetzten eigentlichen (inneren) Reflexion hin. Die äußere Reflexion wendet sich von außen auf den Gegenstand der Betrachtung und setzt ihn eventuell mit Anderem in Beziehung. Sie bedient sich also Methoden, die schon vorliegen. So darf die Logik, insofern sie alle Methodik begründen soll, nicht verfahren. Die Logik muss auch ihre eigene Methodik selbst entwickeln. Insofern die Methodik selbst Komplexität enthält, kann dies nur schrittweise geschehen. Die Methodik muss sich mit dem Gegenstand der Logik entwickeln. Die 'innere Reflexion' ist das Sich-Zeigen von Zusammenhängen und Verbindungen im Gegenstand der Logik. Insofern wir (als Leser) diese Zusammenhänge nachvollziehen in ihrer Zwangsläufigkeit, vollziehen eigentlich wir diese innere Reflexion, die sich uns in der Sache aufdrängt und insofern leicht metaphorisch als die innere Reflexion der Sache selbst angesehen werden kann. Die Sache selbst ist ja unser reiner Geist, auf dessen Gerüst von Bestimmungen wir uns in der Logik konzentrieren. Abhalten müssen wir die äußere Reflexion, da wir so den Gegenstand nicht in seiner Ordnung im Gefüge der Bestimmungen mit den recht verstandenen Beziehungen zu anderen Bestimmungen begreifen, sondern aus diesem Gefüge aussteigen und mehr oder weniger assoziativ und willkürlich versuchen, diesen Gegenstand (d.h. einige der Bestimmungen des reinen Geistes) zu erfassen. Um dieser Unterscheidung willen trennt Hegel auch seine Darstellung im Buch *Wissenschaft der Logik* in die Darstellung der Entwicklung des Gegenstandes selbst (in den Hauptabschnitten) und Anmerkungen, in denen auf unsere Neigung zur äußeren Reflexion und denen sich dort anbietenden Gedanken eingegangen wird. Die Philosophie – die wesentlich Wissenschaft der Logik ist – ist objektiv (wie 'die Sache selbst') und sie ist demonstrativ (nicht intuitiv) in ihrer Methode.

Eine zweite Schwierigkeit (neben der Abhaltung der äußeren Reflexion) betrifft das Verhältnis der Bestimmungen zu den Wörtern der (deutschen) Sprache, welche sie ausdrücken. „In alles, ..., hat sich die Sprache eingedrängt“ [I, 20]. Teilweise bietet die natürliche Sprache – hier zunächst das Deutsche – Ausdrucksformen an, welche möglichst wenig in die Irre führen. Gegenstand der Wissenschaft der Logik sind indessen nicht Worte, sondern Bestimmungen des reinen Geistes, die – bestenfalls – den Kern der Bedeutung einiger Ausdrücke ausmachen. Diese Bedeutungen (Begriffe in einem Verständnis des Wortes „Begriffe“) hängen nicht von der Sprache ab. Ausdrucksformen können sich darin unterscheiden, wie angemessen sie das System des reinen Geistes darzustellen in der Lage sind. Das Buch *Wissenschaft der Logik* stellt eine Bemühung einer angemessenen sprachlichen Darstellung der

Logik und Dialektik dar, es geht allerdings nicht um eine Lexiographie des Deutschen oder irgendeiner anderen Sprache. Auch eine eigens entwickelte philosophische Terminologie kann schaden, indem sie durch die neue Wortwahl Inhalte in die Logik von außen hineinschmuggelt. Hegel betont deswegen immer wieder die Schwierigkeiten des angemessenen Ausdrucks. Da die Bestimmungen des reinen Geistes immer schon vorliegen, sollten wir erwarten, dass sich die natürlichen Sprachen auch ihnen – zumindest schrittweise im Prozess der Selbstaufklärung des Geistes – angepasst haben.³⁵

Während es auf der einen Seite mit „Aufheben“ einen Ausdruck in der deutschen Sprache gibt, der einen wichtigen Aspekt des Prozesses der Komplexitätsanreicherung erfasst, fehlt eine adäquate sprachliche Fassung des 'spekulativen Satzes', der Schaltstelle der Dialektik! An dem Punkt, an dem sich Erkenntnisse wie „Das Sein ist das Nichts“, „Etwas ist das Andere“ usw. finden, und an dem auch Hegels Verständnis von 'Widerspruch' als der Identität und Nicht-Identität zweier Bestimmungen hängt – gerade hier fehlt eine angemessene Ausdrucksform. Eigentlich bedürfte die Wissenschaft der Logik hier eine neuen (spekulativen) Kopula, denn Hegel betont ausdrücklich, dass die Konjunktion von „A ist B“ und „A ist nicht B“ das Ziel doppelt verfehlt: zum einen zerlegt sie die spekulative Einsicht in zwei Teilbehauptungen, wo es auf deren Zugleich ankommt, zum anderen ersetzt sie die dialektische Negation durch die Satznegation. Der spekulative Satz hat auch nichts mit informativen Identitätssätzen (wie „Der Keksdieb war das Krümmelmonster“) gemeinsam: Wenn man sie – wie Frege – durch die Unterscheidung zwischen Referenz und Bedeutung erläutert, scheitert dies schon daran, dass in der Wissenschaft der Logik, wo es ausschließlich um Bedeutungen (den Gehalt der Bestimmungen, die selbst den Kern der Bedeutung entsprechender sprachliche Ausdrücke ausmachen) zu tun ist, es keinen Sinn macht, den Gehalt von SEIN von der Referenz von SEIN zu trennen. Es gibt zwar einen epistemischen Gegensatz zwischen dem Gehalt, wie er an sich ist, und dem Stand seiner Setzung (im dialektischen Prozess), doch verschwindet dieser Gegensatz mit dem Prozess, während die spekulativen Sätze als „Wahrheit ein für allemal zugrunde lieg[en]“ [I, 86]. Offensichtlich kann der spekulative Satz weder eine Bedeutungsgleichheit der Ausdrücke „Sein“ und „Nichts“ meinen, noch deren Äquivalenz. Insbesondere sieht der Satz auch nur wie ein informativer Identitätssatz aus, da zugleich mit der Identität die Nichtidentität behauptet wird, also die Falschheit des Identitätssatzes. Es gilt allgemein, „daß der Satz in Form eines Urteils, nicht geschickt ist, spekulative Wahrheiten auszudrücken“ [I, 93, vgl. E, 178].

35 Hegel sieht hier interessanterweise auch (schon) den Sonderstatus von Definitionen. Definitionen können innerhalb der Sprache nicht kritisiert werden, da sie als Festlegungen (konstitutive Regeln) die Sprache mit konstituieren. Wir können aber Definitionen – und damit einen bestimmten Stand der Sprache – verwerfen, wenn wir sie als unangemessen ansehen. Hegel stellt fest, dass es in der Philosophie, wenn es um Definitionen geht, dann „um *bewährte*, d.h. Solche Definitionen, deren Inhalt nicht bloß als ein vorgefundener aufgenommen, sondern als ein im freien Denken und damit zugleich in sich selbst begründeter erkannt wird“ [E, 210].

An der Schaltstelle der Dialektik finden wir damit einen Gehalt, den wir objektiv erfassen (sollen) und zugleich nicht adäquat sprachlich ausdrücken können!³⁶

Die naive Frage, die sich bei Hegels Betonung des objektiven Charakters des Prozesses des Verweisens der Bestimmungen auf einander – und seiner Metaphorik, die den Bestimmungen selbst entsprechende Handlungen zuweist – schnell stellt, lautet einfach: Wo sind wir Leser bei all diesem mutmaßlich objektiven Geschehen in der Logik? Die Logik muss verstanden werden als eine Betrachtung einer Entwicklung im (reinen) Geist, der wir selbst sind. Im Nachvollziehen des Zusammenhanges des Gerüsts des reinen Geistes nehmen wir zwar Abstand von den so thematisierten Bestimmungen, doch ist die Rede von den Aktivitäten in der Logik gestattet als verdeckte Rede von mentalen Akten, die letztlich wir vollziehen. Die Zuschreibung dieser Aktivitäten an die Bestimmungen selbst soll allein betonen, dass wir hier mit logischem Zwang denken, im Unterschied sogar zur theoretischen Reflexion, die immer noch einen subjektiven Charakter hat. Auch Hegels vermeintlicher Akteur 'der Begriff' meint kaum ein einzelnes Wort oder dessen Bedeutung, sondern den Geist (uns insofern wir auch reinen Geist besitzen) in seinem (unserem) allgemeinen Vermögen zu bestimmen und Bestimmungen zu haben. Die 'Sache bewegt sich' heißt, dass unser Geist, sofern er reiner Geist ist, das System seiner Bestimmungen nachzeichnen kann. Insofern wir dem – für Hegel einen, zwangsläufigen – Pfad durch das System der Bestimmungen folgen, werden wir von ihren logischen Relationen gezwungen, genau so zu gehen. Dass wir als subjektives, persönliches Bewusstsein die Sache in ihrer Bewegung 'schauen' heißt, wir vollziehen zwangsläufige Übergänge nach. Man stelle sich hier den transzendenten Rahmen als ein Netz von Knoten mit Querverbindungen vor! Es geht um unsere mentalen Akte, doch in ihrer objektiven Funktionsweise (d.h. ohne biographisches Beiwerk). Die Übergänge und Abfolgen, welche sich in der Dialektik ergeben, sind transsubjektiv: jeder muss sie so machen, insofern macht sie die Sache. Unser propädeutisches oder innehaltendes subjektives Reflektieren auf diesen Prozess ist die äußere Reflexion. Wir machen uns mit ihr den Prozess noch einmal verständlich. Das Buch *Wissenschaft der Logik* drückt den Prozess in Sprache aus.

Der transsubjektive Charakter der *Wissenschaft der Logik* steht auch hinter Hegels – berühmter, berühmter – Bemerkung, hier werde 'der Geist Gottes' ausgedrückt; sofern Gott ein reines Geistwesen ist, erkennen wir, sofern wir das Gerüst des reinen Geistes erkennen, unabhängig von dessen empirischen Gebrauch ('vor der Schöpfung') den Geist Gottes.³⁷

36 Behelfsweise könnte man eine spekulative Kopula einführen: Das SEIN IST das NICHTS.

37 Interessant an dieser Bemerkung ist m.E. weniger die Hybris Hegels als der Umstand, dass sich hier scheinbar eine extreme Form der Abtrennung unserer geistigen Vermögen von unserer Leiblichkeit ausdrückt. Wenn wir mit Gott (einem – abgesehen von der Inkarnation des Sohnes in Jesus – offensichtlich nicht körperlichen Wesen) das Gerüst des reinen Geistes *teilen*, dann kann der reine Geist nicht körperlich infiziert sein. Zum Bestand der Bestimmungen können dann keine gehören, die wesentlich auf Leiblichkeit verweisen. In der *Wissenschaft der Logik* tritt zwar später das LEBENDIGE INDIVIDUUM auf [vgl. II, 474ff.], das Leiblichkeit besitzt, allerdings

Die größte Schwierigkeit, die sich mit der Wissenschaft der Logik verbindet, ist ihr Charakter als Begriffslogik, als Logik, welche Beziehungen von Bestimmungen/Begriffen und nicht von Urteilen oder Sätzen entfaltet. Wenn in der Wissenschaft der Logik der Gehalt der Bestimmung SEIN entfaltet wird, dann ist die Bestimmung SEIN Gegenstand der Untersuchung und nicht ein Urteil oder ein Satz „Hier ist Sein“ oder „Ich denke an das Sein“ oder etwas dieser Art. Urteile und Sätze haben eine komplexe Struktur. Würde sich die Wissenschaft der Logik Urteilen oder Sätze bedienen, müsste deren komplexe Struktur analysiert werden. Damit wäre Hegels Absicht, die dabei benötigten Bestimmungen in ihrem Zusammenhang (erst) zu entfalten, unterlaufen und mutmaßlich gescheitert. In der Wissenschaft der Logik denken wir Bestimmungen (die Bedeutungen von Ausdrücken), die in ihrem Gehalt aufeinander, nicht auf Urteile/Sätze, in denen sie vorkommen, verweisen. Dazu muss es möglich sein, allein Bestimmungen (also nicht-propositionalen Gehalt) zu denken (in irgendeiner Weise nicht-propositional mental zu repräsentieren). Dies ist mehr als umstritten und wird – bis auf wenige Ausnahmen³⁸ – von Neukanthianern genauso wie in der Analytischen Philosophie bestritten. Ohne diese Annahme lassen sich die Operationen der Dialektik nicht verstehen. Lässt man die Annahme der Begriffslogik fallen, müsste die ganze Dialektik, so man sie denn verstanden hat, neu formuliert werden, was immer dies hieße und was dann noch von ihr übrigbliebe. Diese gewaltige Annahme muss – mit Hegel – gemacht werden, wenn man einen Zugang zum Anliegen und zum Ansetzen von Hegels Dialektik finden will.

§6 Versatzstücke der Methode

Die Klärung der Methode der Logik gehört selbst zum Gegenstand der Logik. Mit den Bestimmungen des reinen Geistes muss somit zugleich die Dialektik geklärt werden als die Methode, welche den Zusammenhang dieser Bestimmungen entfaltet, „denn die Methode ist das Bewußtsein über die Form der inneren Selbstbewegung ihres Inhalts“ [I, 49].

sagt Hegel die „Idee des Lebens für sich ist frei von jener vorausgesetzten und bedingenden Objektivität“ [II, 472].

Hier haben sich die Wege der kantisch-hegelschen Transzendentalphilosophie und der partiell naturalistischen Rekonstruktion der menschlichen Kognition in ihrer auch evolutionären und verkörperten Einbettung in die Wirklichkeit getrennt. Interessant ist diese Bemerkung des Weiteren als Hegel hier Gott eine Natur zuschreibt, über die selbst dieser scheinbar nicht verfügen kann!

38 Eine Ausnahme ist Roderick Chisholms Theorie einer nicht-propositionalen *de se* Theorie des Selbstbewusstseins (in *The First Person*). Aber selbst dort haben die Selbstzuschreibungen nicht nur das Subjekt als dasjenige Relatum, dem sie zugeschrieben werden, sondern auch zumindest solche Strukturen wie: Meinen-X, Sehen-Y, X-sehende usw. Da, wo in den Kognitionswissenschaften von nicht-propositionalen Inhalt gesprochen wird, wird damit immer auch nicht-begrifflicher Inhalt gemeint! Hegels psychologisches Vokabular am Anfang der Seinslogik wird nicht klarer, wenn später der Zugriff auf Bestimmungen als „ein übersinnliches, *innerliches Anschauen*“ [II, 553] bezeichnet wird.

Die Methode bedient sich dabei mehrerer Grundoperationen:

1. *Kontrastierung*: Der Gehalt einer Bestimmung wird entfaltet durch in Beziehung setzen zu anderen Bestimmungen. Der semantische Gehalt verweist selbst auf solche weiteren Bestimmungen, von deren Gehalt der Gehalt der betrachteten Bestimmung abgegrenzt wird. Die Kontrastierung besitzt ein Moment des Abgrenzens, Negierens des anderen Gehaltes, auf den der betrachtete Gehalt verweist.
2. *Komplexitätssteigerung*: Das, was an Gehalt betrachtet wurde, muss als Gehalt einer (neuen) Bestimmung gesetzt werden. Lag eine Beziehung zwischen zwei Bestimmungen vor (etwa wie zwischen dem SEIN und dem NICHTS), so kann der Fortschritt darin liegen, diese Beziehung zwischen zwei Bestimmungen nun als eine Beziehung von Momenten im Gehalt einer komplexeren Bestimmung zu setzen (etwa als das ETWAS mit seinen Momenten AN-SICH-SEIN und SEIN-FÜR-ANDERES).

Die erste Methode erinnert an die Methoden der OLP. Die zweite Methode erinnert an die Methode der ‚Explikation‘ Carnaps bzw. an ‚kreative Synthese‘ als zweitem Schritt der Begriffsanalyse im Sinne Russells.

Bei diesen Grundoperationen geht es immer darum, dass der zu entwickelnde Gehalt in eine Bestimmung gesetzt wird. Das, was der Sache nach (‘an sich’) da ist, muss gesetzt werden. Ein Erkenntnisideal im Fortgang der Wissenschaft der Logik ist somit Explikation. Zugleich wird die Vollständigkeit der Explikation und Darstellung angestrebt. Die Wissenschaft der Logik versteht Wahrheit als „Identität“³⁹, was besser verstanden werden kann als Authentizität: eine Bestimmung hat in der vollen Entfaltung ihres Gehaltes den ihr zukommenden Platz im System der Bestimmungen gefunden; sie kann nicht mehr kritisiert werden, sofern sie Inhaltsmomente abdecken wollte, die besser mit anderen Bestimmungen gefasst werden. Das heißt Bestimmungen „zu reinigen“ und zur „Wahrheit zu erheben“ [I, 27].

Die Operationen folgen nicht beliebig aufeinander. In der Dialektik prozessiert „immanenter Zusammenhang und Notwendigkeit“ [E, 173]. Das erlaubt, die äußere Reflexion abzuhalten und – nur leicht metaphorisch – von der „Überwindung des Gegensatzes von Subjekt und Objekt“ in der Wissenschaft der Logik zu sprechen. Insofern die Methode den Gehalt von Bestimmungen entfaltet, lässt sie sich auch als Bedeutungsanalyse (‘Begriffsanalyse’, ‘conceptual analysis’) verstehen und weist Ähnlichkeiten zu semantischen Analysen in der Analytischen Philosophie auf. Der Fortgang in der Wissenschaft der Logik weist mit der Komplexitätssteigerung (scheinbar) synthetische Momente auf, allgemein muss allerdings festgestellt werden, etwa für das Aufstellen eines spekulativen Satzes als Einheit zweier Bestimmungen:

39 „Übereinstimmung eines Inhaltes mit sich selbst“ [E, 86] und öfter. Damit dies eine nicht-triviale Charakterisierung wird, muss man sich fragen: Wie könnte ein Inhalt nicht mit sich übereinstimmen? (Wittgenstein würde – nicht nur an dieser Stelle – einen philosophischen sprachlichen Unsinn attestieren.)

Die Deduktion ihrer Einheit ist insofern ganz analytisch; wie überhaupt der ganze Fortgang des Philosophierens als methodischer, d.h. als notwendiger nichts anderes ist als bloß das Setzen desjenigen, was in einem Begriff schon enthalten ist. [E, 188]

In beiden Grundoperationen ist auch die hegelsche 'bestimmte Negation' präsent, da eine jeweilige weitere Bestimmung als Kontrast – und insofern negativ – der betrachteten Bestimmung entgegentritt. Die mit der bestimmten Negation verbundene Setzung knüpft an den Gehalt, der auf sie verweist, geht aber nicht darin auf, denn sonst würde die Entwicklung des Bestimmungsgefüges an dieser Stelle abbrechen. Die bestimmte Setzung besitzt somit einen Gehaltsüberschuss von nicht entfaltetem Gehalt, welcher die weitere Entfaltung des Gehaltes bestimmt.

Die Übergänge erfolgen auf keinen Fall angestoßen durch äußere Reflexion, sondern:

- (i) weil eine Bestimmung in ihrem Gehalt auf andere verweist, wie das SEIN als bloße Bestimmung verweist auf die Existenz einer entgegengesetzten Bestimmung, da Bestimmungen nie als einzelne vorliegen, sondern nur als System – dies ist eine notwendige Wahrheit über Bestimmungen,
- (ii) weil eine jeweilige Bestimmung bzw. ein Bestimmungsbestand kritisiert werden kann, sofern (noch) nicht das Gesamt des transzendentalen Rahmens ausgedrückt wird.

Die Bewegung endet mit dem vollständigen System der Bestimmungen und damit der vollständigen Klärung, wie sich welche Übergänge ergeben. Das heißt: es geht um die Bestimmung der Gesamtheit des transzendentalen Rahmens, in der alle Bestimmungen ihren Platz haben – und nicht um die Bestimmung eines Gegenstandes bezüglich dessen alle Bestimmungen nur transitorisch und bis auf die letzte falsch sind. Der transzendente Rahmen ist einer, aber in sich komplex. Dass der 'Begriff' noch nicht 'zu sich gekommen' ist, heißt also, dass das Gesamt der Bestimmungen noch nicht nachgezeichnet, alle Implikationen des Gehaltes noch nicht entfaltet worden sind. Entsprechendes gilt für die positiven Formulierungen, wie „der Begriff kehrt in sich zurück“ usw.

In beiden methodischen Grundoperationen findet sich eine Form von Negativität: als Entgegensetzung oder als Kritik. Das Negative spielt in der Wissenschaft der Logik eine positive Rolle. Es spielt die entscheidende Rolle. Die allgemeine Negation als Satznegation, wie sie aus der Aussagenlogik bekannt ist, kann, da doppelte Negation zur ursprünglichen Aussage äquivalent ist, keinen Fortschritt zustandebringen. Eine Negation der Negation, die sich wie die Satznegation verhält, führt nur in einen Zirkel zurück zur Ausgangsbestimmung. Was 'bestimmte Negation' meint, ist die Negation eines besonderen Inhaltes (Gehaltes einer Bestimmung) in dessen Einseitigkeit (nicht entfaltetem Gehalt). Bestimmte Negation ist bestimmt bezogen auf ihren Ursprung, den Anlass ihres Auftretens. In dieser Bestimmtheit bewahrt sie noch den Bezug auf die von ihr negierte Bestimmung.

Die Dialektik hat ein positives Resultat, weil sie einen bestimmten Inhalt hat oder weil ihr Resultat wahrhaft nicht das leere, abstrakte Nichts, sondern die Negation von

gewissen Bestimmungen ist, welche im Resultate eben deswegen enthalten sind, ... [E, 176f.]

Indem das Resultierende, die Negation, bestimmte Negation ist, hat sie einen Inhalt. Sie ist ein neuer Begriff, aber der höhere, reichere Begriff als der vorhergehende; denn sie ist um dessen Negation oder Entgegengesetztes reicher geworden, enthält ihn also, aber auch mehr als ihn, und ist die Einheit seiner und seines Entgegengesetzten. [I, 49]

Auch die dann erfolgende Negation der Negation ist eine bestimmte Negation der bestimmten Negation, die nicht zirkelhaft auf die zunächst bestimmt negierte Ausgangsbestimmung zurückführt, sondern – quasi durch diese hindurch – zu einer neuen Setzung führt, da bestimmte Negation, die sowohl Gehalt von der Ausgangsbestimmung als auch von der ersten bestimmten Negation enthält. Diese Vorgänge des bestimmenden Entgegensetzens machen, insofern mit ihnen jeweils neuer Gehalt in den Fokus gerät, den Motor der Bewegung in der Sache aus.

Die Gegensätze zwischen einer Bestimmung und einer anderen Bestimmung, welche die bestimmte Negation der ersten ist, haben einen quasi-konträren Charakter. [„quasi-konträr“, da es nicht um einen Gegensatz von Urteilen/Sätzen, sondern um den von Bestimmungen (also nicht-propositionalen Gehalten) geht.]

Die beiden Bestimmungen schließen sich wechselseitig aus, insofern nicht beide zugleich den gerade betrachteten, zu untersuchenden Gehalt besitzen. Die beiden Bestimmungen können jedoch auch bezüglich des gerade betrachteten, zu untersuchenden Gehaltes unzureichend sein (in 'Unwahrheit' sein). Des Weiteren folgt aus ihrem Gegensatz – analog zum konträren Gegensatz bei Urteilen/Sätzen, der auch mehr Inhalt besitzt als die Kontradiktion⁴⁰ – ein quasi-kontradiktorischer Gegensatz, das, was Hegel als 'Widerspruch' begreift. Insofern die Bestimmungen, die in quasi-konträre Gegensätze geraten, erhalten bleiben müssen (in ihrer Rolle im System der Bestimmungen), entfällt die Option, den Gegensatz durch das Fallenlassen mindestens einer der Bestimmungen aufzulösen. Die Dialektik verfolgt nicht das Ziel, die Bestimmungen oder deren Gegensätze zum Verschwinden zu bringen, also ob die Widersprüche nur die jeweils auftretenden aber aufzulösenden Verirrungen der Entwicklung wären (wie Hegel dies wohl Fichte vorwirft). Die Gegensätze werden nur in der Hinsicht aufgehoben, als sich in einer komplexeren Bestimmung der Gegensatz, der zwischen zwei vorausgehenden Bestimmungen auftrat, als Gegensatz zwischen Gehaltsmomenten dieser komplexeren Bestimmung adäquater setzen lässt. Der dreifache Charakter des Aufhebens ergibt sich aus den Erfordernissen des Prozesses der Gehaltsentfaltung:

- (i) Würden die bisherigen Bestimmungen einfach nur verschwinden, läge keine Entfaltung der Gehaltsbeziehungen zwischen Bestimmungen vor, wir wechselten einfach nur das Thema; die bisherigen Bestimmungen müssen also bewahrt werden (in einem Sinne von „Aufheben“).

40 „x ist gelb“ und „x ist rot“ stehen auch in einem kontradiktorischen Gegensatz, obwohl der Gegensatz zwischen GELB und ROT quasi-konträr ist, da „x ist rot“ enthält „x ist nicht gelb“ (in derselben Hinsicht, zur selben Zeit etc.).

(ii) Würden die bisherigen Bestimmungen in ihren bisherigen Beziehungen belassen, läge auch keine Entfaltung der Gehaltsbeziehungen zwischen Bestimmungen vor, wir hätten keinen Fortschritt im Entwickeln solcher Verweise von Bestimmungen aufeinander; die bisherigen Bestimmungen müssen also in etwas Komplexeres eingebettet werden (in einem Sinne von „Heben auf“ ein höheres Level).

(iii) Würden wir bei den bisherigen Bestimmungen verweilen, ergäbe sich wiederum kein Fortschritt, zumal die bisherigen Bestimmungen in ihrem Anspruch, eine vollständige Gehaltsentfaltung zu liefern, kritisiert wurden; die bisherigen Bestimmungen müssen also in diesem ihrem bisherigen Geltungsanspruch zurückgewiesen werden (in einem Sinne von „aufgehoben“).

Aufheben in diesem dreifachen Sinne kann man verstehen als eine weitere Erläuterung der Binnenstruktur der Grundoperation der Komplexitätssteigerung.

§7 Einige Bemerkungen zum Anfangsproblem in der *Wissenschaft der Logik*

Hegel widmet der Frage, womit der Anfang gemacht werden soll, besondere Aufmerksamkeit. Aus der Perspektive eines Verständnisses, das in der *Wissenschaft der Logik* die Erläuterung des System der Bestimmungen des reinen Geistes sieht, mag dies zunächst verwundern. Denn insofern dieses System – immer schon – objektiv vorliegt, muss es ganz gleichgültig sein, von woher man in dieses einsteigt. Von jedem Zugangspunkt aus, müssen sich, sofern die Zusammenhänge überhaupt zwingend sind, alle weiteren Bestimmungen ergeben. Die Schwierigkeit des beliebigen Einstieges ergibt sich jedoch aus dem Darstellungsproblem (je gehaltvoller der Einstieg ist, umso schwieriger wird es sein, hier die äußere Reflexion, in deren Restriktion wir uns üben müssen, fernzuhalten) und der Themensetzung: wird die Methode mit der Sache entwickelt, bringt ein Einstieg vor den einfachsten Bestimmungen die Schwierigkeit mit sich, Komplexitäten der Methode zu verwenden, die noch nicht eingesehen werden konnten. Daher empfiehlt sich der Anfang mit den einfachsten Bestimmungen. Es ist dies auch kein Anfang ohne jegliches objektives Wissen. Wir haben hier schon den transzendentalen Ansatz im Rücken. Wir wissen – mit oder ohne Phänomenologie des Geistes – dass wir das Level des reinen Geistes jenseits des Bewusstseinsgegensatzes betreten (wollen). Wir wissen (objektiv), dass es um das Erfassen einer Vielheit von Bestimmungen geht, die an sich einen bestimmten Gehalt haben. Wir haben als reiner Geist die uns (objektiv) von uns in unserem Selbsterkennen – wie wir es eventuell aus der Phänomenologie des Geistes kennen – auferlegte Pflicht, die reinen Bestimmungen des Geistes erkennen zu sollen.

Bevor einzelne Bestimmung genauer entfaltet wurden, ist auch der Anfang nicht vermittelt, sondern unmittelbar. Selbst das Nichtvermitteltsein ist nicht Gegenstand, da man zu dessen Verständnis die Idee der vermittelten Gehaltsbestimmung verstanden haben muss. Der Anfang präsentiert sich unvermittelt, aber nicht als unvermittelt: weder Vermittlung noch Unmittelbarkeit sind am Anfang gesetzt. Der Gehalt, der am Anfang auftritt, bleibt erhalten. Er wird in seiner Entfaltung erkannt.

Der Anfang wird auch nicht analysiert im Sinne eines Zerlegens in seine Momente, so müsste der Anfang schon als komplex angesetzt sein. Der Anfang wird nur analysiert im Sinne eines Entfaltens des Umstandes, dass noch kein komplexer Gehalt angesetzt wird.

Am Anfang steht ein erster, noch nicht weiter entfalteter Gehalt – dies ist das reine SEIN. Am Anfang haben wir noch keinen besonderen Gehalt erfasst – an sich ist das SEIN nicht unmittelbar, sondern auf anderen Bestimmungen bezogen und nur im Rahmen des Systems der Bestimmungen ist es SEIN: reine Positivität. Am Anfang tritt das SEIN allein im ersten Schritt, als Verweis auf Gehalt überhaupt, unvermittelt auf, da wir eine Bestimmungen denken wollen, ohne schon sagen zu können, welchen Gehalt sie haben soll. Das SEIN ist diese gehaltslose Bestimmung.

Die Reflexion auf die Natur des Anfangs, welche die besondere Rolle des SEIN erläutert gehört allerdings ausschließlich zur äußeren Reflexion. Sie erleichtert die Darstellung, vor allem, wenn es um die Darstellung in einem Buch, wie der *Wissenschaft der Logik* geht. In der Sache werden für den Anfang vorausgesetzt:

- (i) der Prozessgedanke, der als Selbstaufklärung des Geistes zumindest schon bis hierhin führte,
- (ii) die Behauptung der Notwendigkeit der Entwicklung, welche so die äußere Reflexion fernhält,
- (iii) die Behauptung, dass der Prozess des Entfaltens des Gehaltes der Bestimmungen sich selbst weiter treibt.

Es soll ein Anfang gemacht werden, ein Schritt zur völligen Selbstdurchsichtigkeit des reinen Geistes. Deshalb tritt ein 'Sollen' als Wissen sollen auf. Am Anfang gilt: „Nur der Entschluß, ..., nämlich daß man das Denken als solches betrachten wolle, ist vorhanden“ [I, 68].

Hegel kennzeichnet die (einsetzende) Dialektik dabei als „die höhere vernünftige Bewegung“ [I, 111]. Welche Rolle spielt die Bewegung für die Dialektik? Die Schwierigkeiten des spekulativen Satzes hängen vielleicht auch damit zusammen, etwas Dynamisches in etwas Statischem fassen zu wollen. Die semantischen Verhältnisse im transzendentalen Rahmen sind statisch. Für eine semantische Erläuterung allein besteht dieses Problem daher nicht. Es zeigt sich hier wieder: Obwohl die Verhältnisse in der Logik statisch sind, geht es in der Dialektik um das Verfolgen der inneren Reflexion als dem Nachvollziehen der Verhältnisse der Bestimmungen. Dies ist ein originär hegelsches Anliegen, indem selbst die Logik ein Teil der Selbstaufklärung des Geistes über sich ist.

Die Dialektik ist der Prozess, in dem sich der reine Geist über die Strukturen im transzendentalen Rahmen der Bestimmungen klar wird, unter Absehung von der äußeren Reflexion.

§8 Einige Illustrationen zu Übergängen in der Wissenschaft der Logik

Nach langen Vorbereitungen beginnt die Seinslogik mit 1½ Seiten, die in ihrer Knappheit sowohl die ersten Bestimmungen als auch die ersten Operationen der Methode vorführen müssen. Es findet sich allerlei irreführendes psychologisches Vokabular (wenn vom 'Anschauen' die Rede ist), und der erste Übergang wird fast sophistisch formuliert: insofern mit dem gehaltslosen SEIN nichts erfasst wird, wird das NICHTS erfasst! Für den spekulativen Satz „Das SEIN IST das NICHTS“ gibt es gegeben die Überlegungen zum Anfang (§7) und zum Thema der Wissenschaft der Logik (§§ 5,6) zwei bessere Erläuterungen.

Die erste knüpft daran an, dass die Bestimmungen an sich immer schon einen Gehalt haben, auf den die Entwicklung Zugriff hat. Etwa so:

- (1) Das SEIN hat keinen besonderen Inhalt, ist aber eine Bestimmung.
- (2) Eine Bestimmung, welche durch die Abwesenheit von besonderem Inhalt definiert ist, ist das NICHTS.

Also: Das SEIN IST das NICHTS.

Die zweite bezieht sich auf den Umstand, dass wir den Level des reinen Geistes und seiner Bestimmungen betreten. Etwa so:

- (1) Wir wissen, dass im reinen Wissen eine Vielheit von Bestimmungen auftritt.
- (2) Wir wollen/sollen deren Gehalt entfalten.
- (3) Es tritt eine voraussetzungslose Bestimmung als bloße Bestimmung auf: SEIN.
- (4) Als Bestimmung muss sie im Gegensatz zu anderen Bestimmungen bestimmt werden.
- (5) Eine konkrete Bestimmung kann einer leeren Bestimmung nicht entgegengesetzt werden.
- (6) Da SEIN keinen besonderen Inhalt hat, bleibt nur: eine entgegengesetzte Bestimmung, die auch keinen besonderen Inhalt hat.
- (7) Da SEIN und die entgegengesetzte Bestimmung beide keinen Inhalt haben, sind sie inhaltsidentisch.

Also: Das SEIN IST das NICHTS.

Genauso zeigt sich dann wie das NICHTS auf das SEIN verweist, insofern NICHTS eben nicht nichts ist, sondern eine Bestimmung, wenn auch eine ohne (besonderen) Inhalt. Dieser Verweisungszusammenhang besteht als semantischer (den Gehalt der beiden Bestimmungen betreffend) immer schon. Leicht metaphorisch kann man sagen: Der Übergang von SEIN zu NICHTS hat immer schon stattgefunden. Wir als Leser vollziehen die semantischen Verhältnisse ja als Übergänge zwischen den Bestimmungen nach (vgl. §5).

Für das WERDEN gilt damit auch, dass der Übergang (genauer: die beiden Übergänge von SEIN zum NICHTS und vom NICHTS zum SEIN) immer schon stattgefunden haben. Es bleibt indessen die Existenz dieses reinen Gegensatzes vor der Entfaltung konkreter Gegensätze. Die Grundoperation der Konträtierung durch

Entgegensetzen tritt hier auf. Das damit vorliegende Entfalten des Gehaltes und die wechselseitige Bezogenheit sind eine Grundstruktur der Dialektik – gesetzt in der Bestimmung WERDEN. In der Bestimmung WERDEN lernen wir insofern tatsächlich etwas über die Methode. Diese Kategorie ist auch der erste Ausdruck des dialektischen Prozesses.

SEIN und NICHTS bestehen nur im WERDEN (liegen immer als der Verweis auf das jeweils andere vor), sind aber eigenständige Bestimmungen – sonst hätte auch WERDEN nicht seinen ausgezeichneten Charakter. WERDEN selbst kann nicht als erste Bestimmung auftreten, da wir hierzu schon die Verweise, die von den ersten beiden Bestimmungen auf die jeweils andere führen, verstanden haben müssen.

Wie tritt hier Negation auf? Das NICHTS tritt auf als bestimmte Negation des SEIN – und zwar nur bestimmte Negation: Setzen der Negation als Bestimmung, sonst nichts. Das NICHTS ist nicht stabil, kann nicht die ganze Wahrheit sein, da es selbst keinen Inhalt hat und sich so nicht gegen das SEIN abgrenzen kann. Eine Rückkehr zum SEIN hilft ebenfalls nicht, da sie wiederum bezüglich einer Gehaltsfassung genau dieselbe Instabilität aufweist. Tatsächlich zeigt sich am NICHTS, dass es nicht keinen Inhalt hat – doppelte Negation! – sondern eine Geschichte seines Auftretens und Abgrenzens: dies ist sein Inhalt. Dieses wird als bestimmt gesetzt: WERDEN. Im Gehalt von WERDEN liegt daher mehr als der bloße Gehalt von NICHTS. Die Negation der Negation ist die bestimmte Negation der bestimmten Negation derart, dass in die bestimmte Setzung die Geschichte der bestimmten Negation eingeht. Dies zeigt sich zuerst am WERDEN.

Hegel bemerkt später, das ETWAS sei „die erste Negation der Negation“ [I, 123], doch widerspricht dies nicht dem gerade erläuterten Befund. Die Negation der Negation zeigt sich am WERDEN, beim ETWAS wird sie thematisch: Beim ETWAS kommt noch die zweite Grundoperation der Dialektik (die Komplexitätssteigerung) vor: Es liegt nicht nur die Negation der Negation vor (als Relation zwischen zwei Bestimmungen), sondern die Negation (hier: das SEIN-FÜR-ANDERE) wird in den Gehalt der Bestimmung aufgenommen und einem anderen Moment des Gehaltes (hier: das AN-SICH-SEIN) als NEGATION entgegengesetzt. Das WERDEN ist noch nicht in methodischer Vollständigkeit die Negation der Negation, hier zeigt sich indessen das Fortschreiten mittels der bestimmten Negation, das als bestimmte Negation der bestimmten Negation nicht zur ersten negierten Bestimmung zurückkehrt, sondern zu einer komplexeren.

SEIN und NICHTS, obwohl sie wechselseitig ineinander übergehen, bleiben als Pole dieser Übergänge (als Momente) im WERDEN erhalten. Da sich die beiden Übergänge in der Richtung unterscheiden (einmal von SEIN zum NICHTS, einmal von NICHTS zum SEIN), bleiben SEIN und NICHTS nicht nur als Momente (unselbstständig) im WERDEN bewahrt, sondern treten auch in einer komplexeren Weise wieder auf, werden auf eine neue Komplexitätsstufe gehoben: als ENTSTEHEN und VERGEHEN. Als komplett selbstständige (isolierte) Bestimmungen können sie nicht mehr auftreten, insofern verschwinden diese also (für den weiteren Prozess). Hier zeigt sich zum erstenmal das dreifache Aufheben in der Wissenschaft der Logik.

Im WERDEN selbst liegt ein Widerspruch: Obwohl das WERDEN eine Beziehung ausdrückt (d.h. dynamische Momente besitzt), ist diese Bewegung doch etwas, das vorliegt. Wie in einer Schale mit rollenden Kugeln die Schale nicht rollt. Das WERDEN weist so eine ruhige Einheit auf. Diese als Bestimmung gesetzt ist das DASEIN. Im DASEIN ist das SEIN aufgehoben als konkretes SEIN. Als konkretes Sein hat DASEIN eine konkrete QUALITÄT. QUALITÄT als bestimmter Inhalt verweist aber auf den Ausschluss anderer QUALITÄT (bestimmter Inhalte), d.h. verweist auf NEGATION (die Aufhebung des NICHTS als konkrete Entgegensetzung). Dieser Doppelcharakter des Bestimmens von (konkretem) Inhalt wird gesetzt als Gehalt des ETWAS.

Beim ETWAS haben wir zunächst die äußere Beziehung auf ANDERES, insofern sich das ETWAS nur abgrenzend in seinem (konkreten) Gehalt bestimmen lässt. In dieser abgrenzenden Beziehung muss das ETWAS zugleich vom ANDEREN zurückkehren, sich damit bestimmend. Komplexitätssteigerung liegt darin, diesen Zirkel zu verinnerlichen. Das ANDERE, der Bezug auf es und das Sichbestimmen dabei muss in das ETWAS, seinen Gehalt, treten. Man kann dabei den Gehalt des ETWAS zum einen als seinen positiven Gehalt betrachten: sein AN-SICH-SEIN (eine aufgehobene Weise des SEIN). Zum anderen hat das ETWAS einen abgrenzenden, negativen Gehalt: sein SEIN-FÜR-ANDERES (bzw. gegen ANDERES), welches das ANDERE am ETWAS selbst ist (in seinem Gehalt). Diese insgesamt so in seinen Gehalt reflektierte Beziehung zum ANDEREN ist die BESTIMMUNG (im Allgemeinen). Die BESTIMMUNG als positive Gehaltserfassung von ETWAS ist Negation der Negation, insofern ETWAS nun in sich bestimmt wurde und die äußere Beziehung auf ANDERES aufgehoben wurde. Der Gehalt als reflektiert gesetzt ist BESTIMMTHEIT, mit dem Anspruch, dass es auch so (an sich) sei, wobei das Thematisieren dieses Anspruches wieder eine äußere Perspektive aufweist, die es gilt, in eine innere Beziehung zu setzen. Dies geschieht mit der immer wieder verwendeten Einsicht in den Prozess der Inhaltsentfaltung, nämlich dass die Bestimmungen – und damit auch die Elemente in der Vielheit von ETWAS und ANDERES – an sich Gehaltsqualitäten (BESCHAFFENHEIT) jenseits der bisher erreichten Fassung derselben (der BESTIMMUNG) haben. BESTIMMUNG und BESCHAFFENHEIT müssen nicht sofort übereinstimmen, das Ideal der Vermittlung besteht jedoch in der Identität der beiden: erkannter BESTIMMTHEIT (eine BESTIMMUNG welche die BESCHAFFENHEIT erschöpft). In der BESCHAFFENHEIT ist SEIN aufgehoben, in der BESTIMMUNG das NICHTS über die NEGATION in bestimmender Funktion.

Diese Verinnerlichung gilt nun jedoch für beide Pole dieser ursprünglichen äußeren Beziehung von ETWAS und ANDEREM. Es zeigt sich: Das ANDERES ist ETWAS ANDERES. Die beiden Pole teilen die innerlich reflektierte Struktur. Sie sind Elemente einer Vielheit von Gleichartigen, abgeschieden voneinander durch die GRENZE zwischen ihnen. Eine Vielheit von Gleichartigen verweist auf eine GRENZE zwischen diesen. In weiterer Entfaltung zeigt sich so das ETWAS durch die GRENZE als das ENDLICHE

In der Bestimmung GRENZ zeigt sich aufgrund dieser Herleitung allerdings sofort eine Spannung: Die GRENZE bedarf eines Gegensatzes. Die (verinnerlichte) Entwicklung (des Gehaltes) der beiden Pole zeigt indessen, dass diese strukturell gleich sind: es gibt keinen Strukturgegensatz: ETWAS (im Allgemeinen) IST ANDERES (im Allgemeinen). Aber nur im Ausgang vom Gegensatz, dass sie sich einander im Gehalt entgegentreten, konnte ihre Strukturgleichheit entwickelt werden – scheinbar muss also der Gegensatz zugleich bestehen, um zu verschwinden. Insofern ist die GRENZE eine Aufhebung des WERDEN.⁴¹ So wie das WERDEN tritt die GRENZE als ein Drittes zu ETWAS und ANDERES. Ihr Gehalt ist: gemeinschaftliche Unterschiedenheit als Negation des unmittelbaren DASEIN. Insofern die GRENZE das ETWAS vom ANDEREN abgrenzt, ist sie SCHRANKE. Insofern aber selbst die Verinnerlichung des Gegensatzes zum ANDEREN verlangt, dass auf dieses ANDERE zunächst zugegriffen wird, muss diese SCHRANKE immer auch überschritten werden, so dass SOLLEN auftritt. Auch hier erfasst eine Bestimmung ein Moment der Methode.

Dieses Klarwerden der Dialektik selbst muss sich in der *Wissenschaft der Logik* fortsetzen, insbesondere, wenn von der, für Hegel, einfachen Seinslogik zur Wesenslogik übergegangen wird, sofern das Wesen immer schon in sich reflektiert ist. Das Wesen ist als „Sphäre des *gesetzten Widerspruchs*“ [E, 235] auch die Abteilung, in welcher 'der Widerspruch' als zentrales methodisches Konzept weiter geklärt werden muss. Dahin zu gelangen, verlangt allerdings den hier vorgestellten Interpretationsansatz ein großes Stück weiter zu verfolgen. Für sich genommen erscheinen Hegels Ausführungen zum Widerspruch, zum *tertium non datur* und zum Schließen eher obskur.

§9 Ausblick

Auch bei Hegels Dialektik muss man ebenso wie bei Kant unterscheiden zwischen einem evtl. sinnvollen Ansatz und Hegels eigener Durchführung. Die Übergänge und Zusammenhänge von Bestimmungen, die Hegel anbietet, haben oft den Charakter wenig zwingender semantischer Assoziationen oder gar sophistischen Gehalt (etwa wenn Ausgeschlossenheit und Nichtsein um der Herleitung eines spekulativen Satzes willen äquivoliert werden). Die Rede vom 'Spekulativen Satz' und von den entsprechenden 'Widersprüchen' selbst muss mutmaßlich herabgestuft werden zu einer Rede von Entgegensetzungen in inhaltlicher Bezogenheit und Differenzierung von Aspekten der Übereinstimmung (Identität) und Nichtübereinstimmung (Nichtidentität).

Des Weiteren kann gerade eine Explikation der Hegelschen Dialektik im Rahmen einer analytischen Transzendentalphilosophie die Frage aufwerfen, ob die sinnwol-

41 Dieser 'Widerspruch' der GRENZE hat wieder etwas Sophistisches, insofern zwischen Strukturgleichheit und Gehaltsgleichheit hin und her gewechselt wird. Es zeigt sich m.E. schon sehr bald im Buch *Wissenschaft der Logik*, dass man Hegels Programm von seiner (rhetorischen) Durchführung bei Hegel selbst trennen sollte. Bezüglich der GRENZE kommt es mir hier nur auf den Hinweis an, dass auch das WERDEN in aufgehobener Gestalt wieder auftreten muss, und dies in der GRENZE tut.

len Anliegen der hegelschen Konzeption nicht besser anders realisiert werden können. Die entwickelte philosophische Logik der Gegenwart kann zur (axiomatischen) Explikation semantischer Beziehungen eingesetzt werden. Dies betrifft sowohl logische Begriffe (wie Identität und Existenz) als auch Axiomaten für Prozesse, sowie die Darstellung semantischer Beziehungen durch Negationsjunktoren und Enthaltenseinsbeziehungen verschiedener Art.

Das ist nicht was Hegel gewollt hat – allein schon, weil er diese Logik(en) nicht kannte, vor allem auch, weil es sich um eine Theorie eines axiomatischen, propositionalen Gerüsts handeln würde, nicht um einer Logik nicht-propositionaler Beziehungen. Dass es keine guten Formalisierungen von Hegels Dialektik – auch keine parakonsistenten – gibt, ist kein Zufall.

Das Meiste spricht dafür, dass eine Rekonstruktion im Kontext gegenwärtiger philosophischer Logik und analytischer Erkenntnistheorie den transzendentalen Rahmen des Geistes besser erfasst als das hier angedeutete hegelsche Programm. Eine propositionale logische Modellierung mag sogar in der ein oder anderen Implementationsform 'psysisch real' ist.

Der Erfolg eines zweiten Blicks auf Hegels Dialektik wäre somit das vorläufige Ergebnis:

- (i) Unter einigen starken Annahmen (wie der Möglichkeit einer reinen Begriffslgik) lässt sich eventuell ein erkenntnistheoretischer Sinn der Wissenschaft der Logik als semantisches Programm entwickeln.
- (ii) Re-interpretiert man das hegelsche Vokabular in diese Richtung lässt sich konsequent ein Teil der Wissenschaft der Logik so lesen.
- (iii) Damit ist nicht gesagt, dass die Gesamtkonstruktion schließlich trägt.
- (iv) Bezüglich der sinnvollen Ziele des hegelschen Programms könnten andere Ansätze vielversprechender sein.

Um all dies fundierter abwägen zu können, müsste sowohl eine entsprechende Gesamtinterpretation der *Wissenschaft der Logik* zunächst einmal vorliegen – mutmaßlich ein monumentales Programm und eine kaum zu bewältigende Aufgabe. Außerdem steht dann noch die philosophiehistorische, intentionalistische oder hermeneutische Frage im Raum, ob diese Interpretation Hegel gerechter wird als die schon vorliegenden Interpretationen, was wiederum auf ein monumentales Programm des begründeten Vergleiches zur Hegel-Literatur verweist.

Das Interesse, sich solche Aufgaben aufzubürden, bleibt wohl proportional schwach zur geringen Wahrscheinlichkeit, dass Hegels Dialektik Einsichten birgt, die sich nicht auch anders exakter – und damit besser und einfacher – fassen lassen.

References

- Åqvist, Lennart (1987). *Introduction to Deontic Logic and the Theory of Normative Systems*. Neapels.
- Batens, Diderik (2000). "A Survey of Inconsistency-Adaptive Logics", in: Batens, D. et al. (Eds.) *Frontiers of Paraconsistent Logic*: Baldock, pp. 49-73
- Beall, JC. (Ed.) (2007). *Revenge of the Liar*. Oxford.
- Bendall, Kent (1979). "Negation as a Sign of Negative Judgement". *Notre Dame Journal of Formal Logic*, XX, pp. 68–76.
- Besnard, Philippe (1989). *An Introduction to Default Logic*. Berlin et al.
- Blackburn, Patrick/de Rijke, Maarten/Venema, Yde (2001). *Modal Logic*. Cambridge.
- Boolos, George/Jeffrey, Richard (1989). *Computability and Logic*. Cambridge, 3rd Edition.
- Bremer, Manuel (2005). *Philosophische Semantik*. Frankfurt a.M.
- (2005a). *An Introduction to Paraconsistent Logics*. Bern et al.
 - (2007). "Varieties of Finitism", *Metaphysica*, 8, pp. 131-48.
 - (2007a). "Believing and Asserting Contradictions", *Logique et Analyse*, 50.
 - (2008). *Conceptual Atomism and Justificationist Semantics*. Frankfurt a.M. et al.
 - (2008a). "Kearns' Illocutionary Logic and the Liar", *History and Philosophy of Logic*, 29, pp. 223-25.
 - (2010). *Universality in Set Theories*. A Study in Formal Ontology. Frankfurt a.M.
 - (2013). "Restall and Beall on Logical Pluralism: A Critique", *Erkenntnis*.
- Cantor, Georg (1887). "Mitteilungen zur Lehre vom Transfiniten I", *Zeitschrift für Philosophie und philosophische Kritik*, 91, pp.81-125, 252-70.
- Cappelen, Herman/LePore, Ernie (2007). *Language Turned On Itself*. The Semantics and Pragmatics of Metalinguistic Discourse. Oxford.
- Chalmers, David (1996). *The Conscious Mind*. Oxford.
- Chisholm, Roderick (1981). *The First Person*. Minnesota.
- Chomsky, Noam (1986). *Knowledge of Language*. New York.
- (2005). *New Horizons in the Study of Language and Mind*. New Edition. Cambridge/MA.

- Coffa, Albert (1993). *The Semantic Tradition from Kant to Carnap*. To the Vienna Station. Cambridge.
- Dauben, Joseph (1979). *Georg Cantor. His Mathematics and Philosophy of the Infinite*. Princeton.
- Davidson, Donald (1968). "On Saying that", *Synthese*, 19, pp. 130-46.
- (1980). 'Deception and Division', in J. Elster (Ed.) *The Multiple Self*. Cambridge, pp. 79-92.
- Dummett, Michael (1963) "The philosophical significance of Gödel's theorem", *Ratio*, 5, pp. 140–155.
- (1981) *Frege. Philosophy of Language*. Cambridge, 2nd Ed.
 - (1991). *The Logical Basis of Metaphysics*. London.
 - (1991a). *Frege. Philosophy of Mathematics*. London
- Fitting, Melvin (2007). *Incompleteness in the Land of Sets*. London.
- Fodor, Jerry (1975). *The Language of Thought*. New York.
- (1998). *Concepts. Where Cognitive Science Went Wrong*. Oxford.
- Frege, Gottlob (1879). *Begriffsschrift*. Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens. Halle a. S.
- (1884). *Die Grundlagen der Arithmetik*. eine logisch-mathematische Untersuchung über den Begriff der Zahl. Breslau
 - (1893). *Die Grundgesetze der Arithmetik*. Band I. Jena.
 - (1971) *Schriften zur Logik und Sprachphilosophie*. Aus dem Nachlass. Hg. v. Gottfried Gabriel. Hamburg.
- Gabriel, Gottfried/Kambartel, Friedrich/Thiel, Christian (Eds.) *Gottlob Freges Briefwechsel*. Hamburg, 1980.
- Gettier, E. (1963). "Is Justified True Belief Knowledge?", *Analysis*, 23, pp.121-23.
- Goodman, Nelson (1965). *Fact, Fiction, and Forecast*. Indianapolis.
- Hallett, Garth (2008). *Linguistic Philosophy*. The Central Story. New York.
- Hallett, Michael (1984). *Cantorian Set Theory and Limitation of Size*. Oxford.
- Hanfling, Oswald (2000). *Philosophy and Ordinary Language*. The Bent and Genius of our Tongue. London/New York.
- Hare, Richard (1952). *The Language of Morals*. Oxford.
- Heck, Richard (2011). *Frege's Theorem*. Oxford.
- Hintikka, Jaakko (1962). *Knowledge and Belief*. An Introduction to the Logic of the two Notions. Ithaca.
- Hirsch, Eli (1993). *Dividing Reality*. New York/Oxford.
- Hoche, Hans-Ulrich/Strube, Werner (1985). *Analytische Philosophie*. Freiburg/München.
- Hoffmann, Joshua/Rosenkrantz, Gary (2002). *The Divine Attributes*. Oxford.

- Hugly, Philip/Sayward, Charles (2006). *Arithmetic and Ontology*. A Non-Realist Philosophy of Arithmetic. Amsterdam/New York.
- Humberstone, Lloyd (2000). "The Revival of Rejective Negation", *Journal of Philosophical Logic*, 29, pp. 331–81.
- Jackson, Frank (1998). *From Metaphysics to Ethics*. A Defense of Conceptual Analysis. Oxford.
- Kalderon, Mark Eli (2005) (Ed.). *Fictionalism in Metaphysics*. Oxford.
- Kaplan, David (1969). "Quantifying In", in: Davidson, Donald/Hintikka, Jaakko (Eds.) *Words and Objections: Essays on the Work of W.V. Quine*. Dordrecht, pp. 178-214.
- Kuhlmann, Wolfgang (1985). *Reflexive Letztbegründung*. Freiburg/München.
- Larson, Richard/Segal, Gabriel (1995). *Knowledge of Meaning*. An Introduction to Semantic Theory. Cambridge/MA et al.
- Lenzen, Wolfgang (1979). "Epistemologische Betrachtungen zu [S4, S5]", *Erkenntnis*, 14, pp.33-56.
- (1980). *Glauben, Wissen und Wahrscheinlichkeit*. System der epistemischen Logik. Wien/New York.
- Leśniewski, Stanisław (1929). "Grundzüge eines neuen Systems der Grundlagen der Mathematik." *Fundamenta Mathematicae*, XIV, pp. 1-81.
- Lewis, David (1986). *On the Plurality of Worlds*. Oxford.
- (1991). *Parts of Classes*. Oxford.
- Linnebo, Øystein (2003). "Plural Quantification Exposed", *Nous*, pp. 71- 92.
- Mahler, Laurence (1968). *Finite Sets*. Theory, Counting, and Applications. Columbus.
- Malcolm, Norman (1942). "Moore and Ordinary Language", in: Schilpp, Paul (Ed.). *The Philosophy of G. E. Moore*. Evanston/Chicago, pp. 345-68.
- (1959). *Dreaming*. London.
- McGinn, Colin (2012). *Truth by Analysis*. Games, Names, and Philosophy. Oxford.
- Meyer, J.-J. Ch./van der Hoek, W. (1995). *Epistemic Logic for AI and Computer Science*. Cambridge
- Parsons, Charles (1976). "Some Remarks on Frege's Conception of Extension", in: Schirn, Matthias (Ed). *Studien zu Frege*. Stuttgart, Vol. 1, pp. 265-77.
- Peacocke, Christopher (2008). *Truly Understood*. Oxford.
- Petersen, Michael/VanArragon, Raymond (2004). (Eds.) *Contemporary Debates in Philosophy of Religion*. Oxford.
- Priest, Graham (1979). "The Logic of Paradox", *Journal of Philosophical Logic*, 8, pp. 219-41.
- (1987). *In Contradiction*. Dordrecht.

- Pylyshyn, Zenon (1984). *Computation and Cognition*. Towards a Foundation for Cognitive Science. Cambridge/MA.
- Quine, Willard Van Orman (1963). *Set Theory and Its Logic*. Harvard.
- Quinn, Philip L./Taliafferro, Charles (1997). (Eds.) *A Companion to Philosophy of Religion*. Oxford.
- Rawls, John (1973). *A Theory of Justice*. London.
- Ricketts, Thomas (2010). "Concepts, objects and the Context Principle", in: *The Cambridge Companion to Frege*, ed. by Michael Potter and Tom Ricketts, Cambridge, pp. 149-219.
- Russell, Bertrand (1902). *The Principles of Mathematics*. Cambridge.
- (1910). *Principia Mathematica*. Vol. I, Cambridge.
 - (1918). *The Philosophy of Logical Atomism*. Reprinted, London, 1956.
 - (1919). *Introduction to Mathematical Philosophy*. London.
- Sartwell, Crispin (1992). "Why Knowledge is Merely True Belief", *The Journal of Philosophy*, LXXXIX, pp. 167-80.
- Schnelle, Helmut (1973). *Sprachphilosophie und Linguistik*. Reinbek.
- Schurz, Gerhard (1991). "Relevant Deduction. From Solving Paradoxes Towards a General Theory", *Erkenntnis* (35), pp.391-437.
- Searle, John (1969). *Speech Acts*. Cambridge.
- Searle, John/Vanderveken, Daniel (1985). *Foundations of Illocutionary Logic*. Cambridge et al.
- Sellars, Wilfried (1963). *Science, Perception and Reality*. London.
- Shapiro, Stewart (1991). *Foundations without Foundationalism. A Case for Second-Order Logic*. Oxford.
- Simmons, Peter (1987). *Parts*. A Study in Ontology. Oxford.
- Stein, Edward (1996). *Without Good Reason*. Oxford.
- Stern, Robert (1999). (Ed.) *Transcendental Arguments*. Oxford.
- Strawson, Peter Frederick (1952). *An Introduction to Logical Theory*. London.
- (1985). *Scepticism and Naturalism*. Columbia.
 - (1992). *Analysis and Metaphysics*. An Introduction to Philosophy. Oxford.
- Stump, Eleonore/Murray, Michael (1999). (Eds.) *Philosophy of Religion*. The Big Questions. Oxford.
- Tennant, Neil (1987). *Anti-Realism and Logic*. Oxford.
- Terman, Folke (1993). *Reflective Equilibrium*. Karlshamn.
- Turing, Alan (1936). "On Computable Numbers with an Application to the Entscheidungsproblem", *Proceedings of the London Mathematical Society*, Series 2, 42.

- von Kutschera, Franz (1989). *Gottlob Fege*. Berlin/New York.
- von Savigny, Eike (1969). *Die Philosophie der normalen Sprache*. Eine kritische Einführung in die „ordinary language philosophy“. Frankfurt a.M.
- Williamson, Timothy (2007). *The Philosophy of Philosophy*. Oxford.
- Winch, Peter (1958). *The Idea of a Social Science*. London.
- Wittgenstein, Ludwig (1953). *Philosophical Investigations*. Oxford.
- (1964). *Remarks on the Foundation of Mathematics*. Oxford.
 - (1974). *On Certainty*. Oxford, 2nd Ed.
- Woozley, A. D. (1952). “Knowing and Not Knowing”, *Proceeding of the Aristotelian Society*, 53, pp. 151-72.

The book aims to set out in which respects concepts are properly studied in philosophy, what methodological role the study of concepts has in philosophy's study of the world, why there are several viable methods of analysis and even conceptual analysis has its place here. Many of the considerations in this book nowadays are placed under the headline 'metaphilosophy'. The book starts with some bold theses in favour of a representationalist theory of meaning and concepts which serve as the background for the discussion in the following chapters. In contrast to paradigmatic ordinary language philosophy the book endorses a representationalist theory of meaning and concepts, thus agreeing with many of its critics in philosophy and the cognitive sciences. In contrast to many of these critics and supposedly the majority of cognitive scientists it endorses the viability of conceptual analysis as one method of philosophy.

The book reflects on Frege's theory of concepts, because Frege's theory of concepts was one strand that inaugurated analytic philosophy. Frege's theory of sentential unity has barely been superseded, and the problems arising from Frege's understanding of concepts are still alive. Frege's theory and the related problems in Frege's logic as in the *Grundgesetze der Arithmetik* (most famously the antinomy known as 'Russell's Paradox' going back to Frege's 'Basic Law V') lead over to considering the proper approach to our concept of logic and the issue of psychological and ontological realism in logic and mathematics.

The central part of the book starts by reconsidering the approach and the idea of ordinary language philosophy and its understanding of conceptual analysis. Although ordinary language philosophy cannot be the whole of analytic philosophy a proper understanding of conceptual analysis turns out to be one part of analytic philosophy. This part starts with a general discussion of ordinary language philosophy, but proceeds then by a methodological overview and attempts to engage in some ordinary language philosophy concerning epistemological topics.